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MATHEMATICAL MODELLING OF A LIVING-ROOM WITH A SOLAR RADIATION SOURCE AND DIFFERENT BOUNDARY CONDITIONS

DZĪVOJAMĀS ISTABAS AR SAULES STAROJUMA AVOTU UN DAŽĀDIEM ROBEŽNOSACĪJUMIEM SKAITLISKĀ MODELĒŠANA

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Introduction

Heat balance in a living-room is very important from the energy consumption angle and available solar radiation has an essential influence on it. For the first approximation this kind of heat transfer can be neglected and only heat conductivity in the boundary constructions and convection in the room under different ventilation conditions observed. Corresponding numerical modelling for a living- room with different boundary constructions, appropriate optimisation of an arrangement of the heater and ventilation system, corresponding heat consumption, temperature and velocity fields and the effect on heat balance and thermal comfort in the room are analysed in detail [1, 2].

The radiation heat transfer from the heater's surface and through the window must be taken into account for more accurate numerical results and physical interpretation. On sunny days, owing to the considerable solar heat transfer through the window, required temperature in the room can be maintained with less heat from the central heating system. The mathematical modelling enables us to include the

solar heat source and radiation heat transfer from the heater in the total heat consumption.

Another important issue to be taken into account is thermal comfort conditions. They are generally affected by many factors, such as velocity of airflows, humidity, absolute temperature and amplitude of the vertical temperature gradient in the room [3]. It is therefore necessary to analyse these factors using different variants of model conditions.

Air exchange also plays a significant role in guaranteeing an oxygen feed for the rooms inhabited by humans: therefore airflows through openings and the ventilation system are to be analysed in variants with different pressure conditions. The greater air exchange rate, however, means not only more fresh air but also greater convective heat losses and increased use of heating. Therefore desire for optimum comfort conditions for human living can conflict with the necessity of heating energy saving and correlation of this factors should be optimized.

Problem Formulation

A living-room with convective boundary conditions on exterior border structures, heater surface temperature and air openings is modelled. A radiation heat transfer model with an additional solar source and various angles of attack is also included. Considered variants help to predict the features of heat transfer process in the room and distributions of characteristic quantities. This approach also characterises the effect of the radiation heat transfer and the conditions of thermal comfort. *ANSYS CFX* software [4] is used for development of 3D mathematical models and numerical calculations.

The calculations have been performed for a room with the dimensions 2.75×4×6 *m*, shown in Fig.1, and filled with air. The window and the wall to the exterior air are modelled using different materials with heat transmittance for the window U_{window}=2.5 $W/(m^2 \cdot K)$ and for the wall U_{wall}=0.35 $W/(m^2 \cdot K)$. Such values are chosen as they equate to a room with an insulated outer wall and ordinary double-glazed window. Heat transmittance for other boundaries (walls to the adjoining rooms and to the corridor) U_{other} is set to 1 $W/(m^2 \cdot K)$. The heat transfer coefficient α from the surfaces is included in the heat transmittance value using expression $\alpha = \left(1/\alpha_{\text{st}} + R\right)^{-1}$ for all solid structures. Here α_{st} is standardised heat transfer coefficients from the surfaces (23.2 $W/(m^2 \cdot K)$ for outside and 8.1 $W/(m^2 \cdot K)$ for adjoining rooms).

It is assumed that the surrounding rooms (upstairs, downstairs, and side rooms) have the temperature (T) of 20 ºC, but the end wall is contiguous with a corridor or a staircase where the temperature is lower (T=15 ºC). The outdoor temperature is set as for winter conditions (T=−10 ºC). Temperature values are used for the setting-up of convective boundary conditions.

Solar radiation through the window is modelled as a heat source on the inner window's surface. Taking into account the measurements of window solar transmittance [4], the solar heat flux density is set to 500 W/m^2 on the inner part of the glass. Angle of attack for solar radiation varies from 30 to 60 degrees, which models different sun altitudes. For the radiation simulation the Monte Carlo model [5] with $2 \cdot 10^6$ histories is used. All objects except the transparent window are modelled as grey bodies with emission ε=0.9.

A small cranny between the window and the wall is created in some variants to model real gaps in old window-frames: however, in the opposite wall a ventilation

opening has been created. Opening boundary conditions with constant pressure and temperatures of -10 ºC and 15 ºC are defined on the surfaces of the crannies and ventilation openings. Pressure difference ΔP between outer wall and the wall to the corridor is set to constant 0 and 1 *Pa* underpressure or overpressure to model different windy conditions. Surface temperature of the heater is set to a constant 50 °C. For all surfaces except openings non-slip boundary conditions are set.

Different features of all considered variants are summarized in Table 1. The first variant, A, represents a room with radiation modelling from the heater's surface but without solar heat transfer through the window. The angle of solar radiation is varied in variants B1-B3, to imply different powers on the window surface and the additional heat source from the heater. Variant C represents a case with a solar source through the window but without additional heat transfer from the heater (adiabatic condition), which allows the simulation of a living- room in winter conditions with a switched-off heating system. For the next three variants a room with an air infiltration is considered using different boundary conditions on the openings – variant D1 without pressure difference ($\Delta P=0$ *Pa*), variant D2 with 1 Pa overpressure in the room and variant D3 with 1 Pa underpressure in the room. The airflows in the room depend both on the convection created by the temperature difference and on the air exchange between the openings in the structures for the last three variants.

To describe the quasi-stationary behaviour of temperature and average turbulent flows traditional differential equations are employed [6]:

- Reynolds averaged Navier-Stokes (RANS) equation and equations for turbulent energy and dissipation;
- continuity equation;
- energy conservation equation.

As the velocities in some regions are high, SST $k-\omega$ turbulence model [4, 7] is used.

3D discretisation was performed with tetrahedral elements of varying size; boundary layers are discretized with smaller prismatic elements. The characteristic size of finite elements is from 20 *cm* in the middle of the room to 0.5 *mm* in the vicinity of the heating element and for the openings in the walls. The total number of elements reaches $7 \cdot 10^5$ depending on geometry. The typical meshes in the middle cross-section of the room and near the heater are shown in Fig. 2.

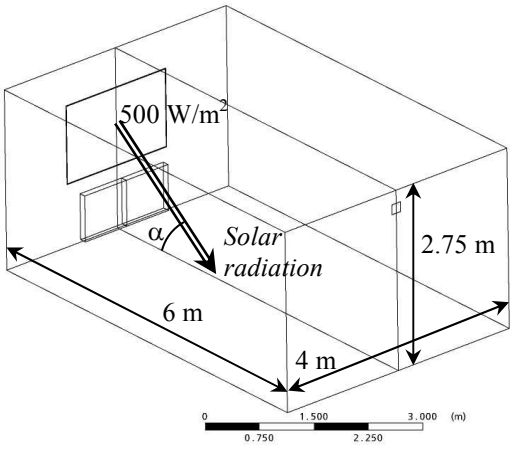


Fig. 1. Layout of a modelled room.

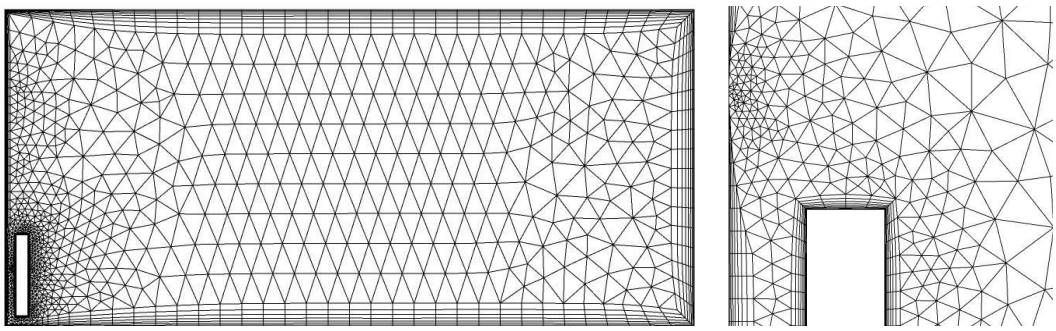


Fig. 2. Typical meshes for the modelled room in the middle cross-section and near the heater.

Table 1. Properties of various variants considered and main results of the modelling.

Properties/results	Variants (variable parameters are shown in bold face)							
	A	B1	B2	B3	C	D1	D2	D3
Angle of attack α (degrees)	-	**30**	**45**	**60**	45			
Solar heat flux density (W/m^2)		500						
Boundary condition on heater	temperature, 50°C				**adiabatic**	temperature, 50°C		
Pressure difference ΔP (Pa)	-					**0**	**1**	**-1**
Heat amount for the heater (W)	225	173	173	178	0	254	232	315
Solar power, W	0	411	327	228	333	326	325	325
Ventilation heat losses (W)	-					98	181	440
Air exchange rate ($1/h$)	-					0.3	1.4	1.4
Average velocity v (cm/s)	2	5	6	5	3	4	6	6
Average temperature T (°C)	24.2	32.5	30.8	28.4	26.0	23.1	26.1	22.4
Vertical temperature difference ΔT (°C)	2.1	1.7	1.7	2.5	1.5	1.5	1.2	2.4

The boundary conditions of the third type (convective) and the low viscosity of air essentially worsen the convergence of an iteration process. Each modelling simulation has therefore been performed in two steps – at first a steady state calculation, then transient state with constant properties and initial conditions from the steady state results. The total time required for one variant calculation with a 3 GHz computer is about 2 days. The calculated heat imbalance between the heater power, the solar source and the losses from the outer surfaces and openings decreases below 3 %.

General results

Results for all nine considered variants are summarized in Table 1 and visualized in Fig. 3. We can consider following result groups:

- the room without a solar source (A);
- solar radiation with different angles of attack α and constant heat flux density (B1 – B3);
- switched-off heating system in winter conditions with a solar source (C);

- inclusion of air infiltration process in the room with a solar source (D1 – D3).
-

It is possible also to extract two significant parts of the results – heat balance of the room (heating powers, ventilation heat losses and air exchange) and thermal comfort conditions (average velocity and temperature, vertical temperature difference).

For the basic variant without a solar source an average temperature in the room is 24.2 °C and there is a characteristic hot air flow at the top of the heater near the window (see Fig. 4a). Emission coefficient ε=0.9 set for the heater's surface in fact is greater than for the real heaters and it demonstrates the maximum potential power of a heater. The considered variants, however, in comparison with other considered variants without radiation modelling [1, 2] show an essential influence of the radiation from the hot surfaces on the total heat balance and this kind of heat transfer cannot be ignored for accurate qualitative results. For the model without radiation modelling, the heat amount taken from the heater is more than 25% less and the average temperature in the room is only 21.6 °C.

Results of the variants B1-B3 with solar sources show that at the small angles of attack the solar radiation power is increased because the sun is shining deeper into the room (Table 1, Figs. 4b-4d and Fig. 5). Thus, the average temperature in variant B1 with a 60° angle of solar source is 28.4 °C, but in variant B3 with a 30° angle it is 4 degrees higher. Heat amount from the radiator for variants B1-B3 is nearly the same and the changes of high temperature level is determined only by solar radiation. Thermal comfort conditions in this situation are not suitable and the thermostatic control of the heating system or installation of a ventilation system is necessary to decrease the temperature in the room.

Results for variant C with a switched-off heating system clearly shows, that the power of solar radiation provides sufficient heat amount for the average temperature 26 °C (Table 1, Figs. 3) and additional heating is not needed. This result is also achieved for the room without air infiltration and shows the maximum potential temperature level in the room. Convection heat transfer process is to be taken into account to model rooms more accurately.

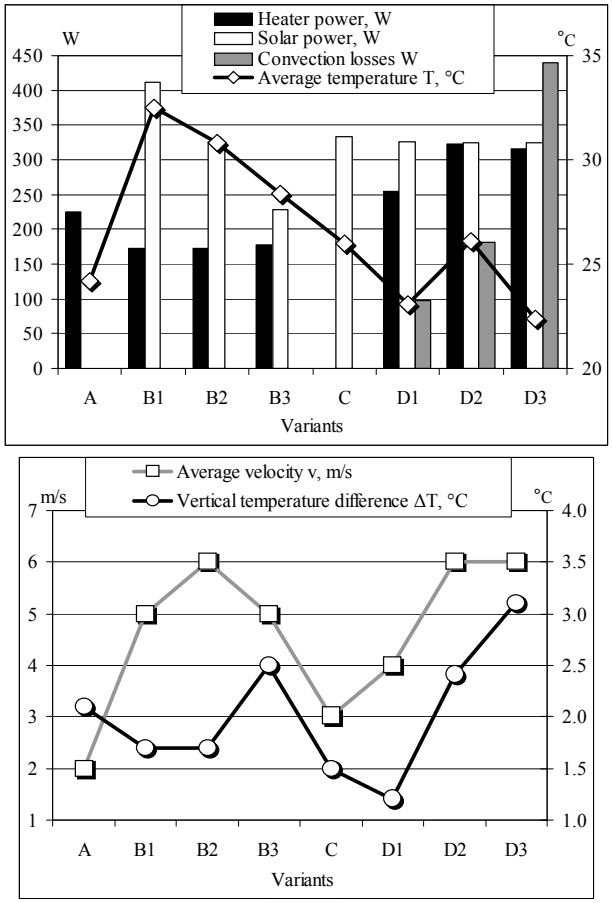


Fig. 3. Heating powers, convection losses, average temperature in the room, vertical temperature difference and airflow velocity in the middle of the room for different variants.

Three different options of air infiltration conditions in the room are considered in variants D1-D3. In cases, when no pressure difference is set between the ventilation opening and the crannies in the window-frame, an insignificant air circulation owing to thermoconvection exists in the room and the air exchange rate is 0.3 1/*h*. This value is minimal for an oxygen inflow needed for human occupancy and does not produce great heat losses (Table 1).

When 1 *Pa* overpressure is set on the ventilation opening (variant D2), air masses with constant temperature of 15 °C are flowing into the room. This creates intensive flow in the middle of the room (Fig.4g) and average velocities in the room are high (Table 1 and Fig. 3). The variant D3 describes the room with underpressure in the room owing to 1 *Pa* pressure set on the crannies in the window-frame, which means active cold exterior air with temperature of –10 °C inflow into the room (Fig.4h). As a result of more intensive airflows heat transfer from the radiator shows a remarkable increase – from 254 **W** without pressure difference to 315 *W* in this variant (Table 1 and Fig. 3). Cold masses from the outside decrease average temperature in the room from 30.8 °C for variants without air infiltration to 22.4 °C.

For both variants with pressure difference 1 *Pa* (D2 and D3) the air exchange rate is 1.4 *1/h*; this causes great convection heat loss and is disadvantageous in energy

terms. The overpressure/underpressure conditions in the room is determined by the meteorology and the heat losses through the openings are not controllable.

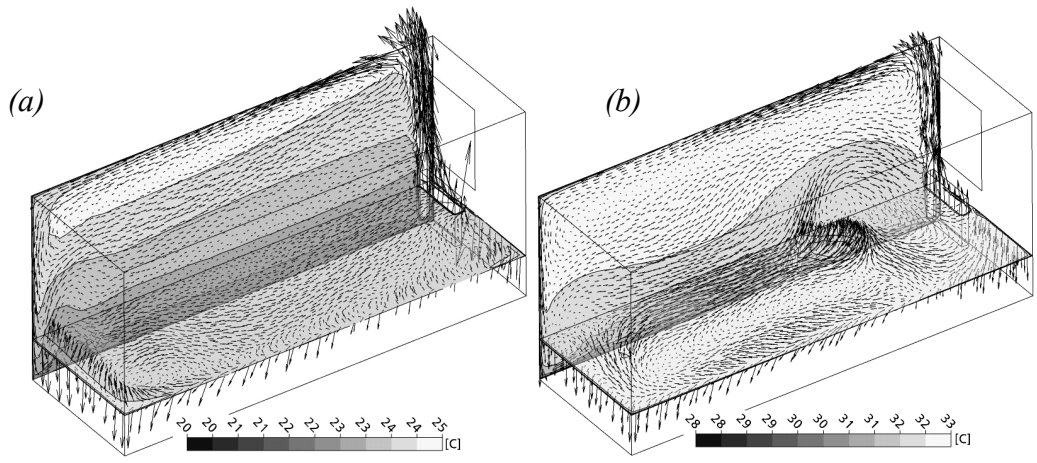
Result analysis

Thermal conditions in the room are connected to the structure of airflow velocities, pressure difference between openings in boundary constructions, temperature distribution and thermal convection: thus, an intensive upward airflow is formed at the top of the heater with maximum velocities up to 60 *cm/s*, and at the same time an average air movement in the whole room does not exceed 5 *cm/s*. In the case of solar radiation, hot regions on the floor produce sizeable convection and increase average velocities in the room. The direction of this movement, however, is horizontal in variants with hot radiator, because of the existence of other big vortices in the room, but almost vertical in the variant C with its switched-off heating system (Fig. 4). In general, the radiation source through the window increases airflow intensity in the middle part of the room (Fig. 3).

A complicated multiple vortex airflow structure in the room with maximum velocity of 50 *cm/s* has been created in the variant with an air infiltration opening without pressure difference (variant D1, Fig. 4f). It is caused by three main factors:

- inflowing air from the corridor with T=15 °C moves downwards near the wall;
- hot air from the alight region (Fig. 5) on the floor moves upwards and the flow splits;
- hot airflow from the heater moves upwards near the window and creates the intensive vortex.

The situation changes with an overpressure of 1 Pa in the room (variant D2) and only one main vortex is formed in the room owing to active air inflow from the corridor (Fig. 4g). As the air masses move through the relatively small ventilation opening, velocities there are very high – up to 1 *m/s*, but air movement in the other part of the room does not exceed 10 *cm/s*. Direction of the airflow vortex in the room changes as the pressure difference conditions change, but also in the case of underpressure (variant D3) air velocities near the hot region on the floor are about 30 *cm/s*.



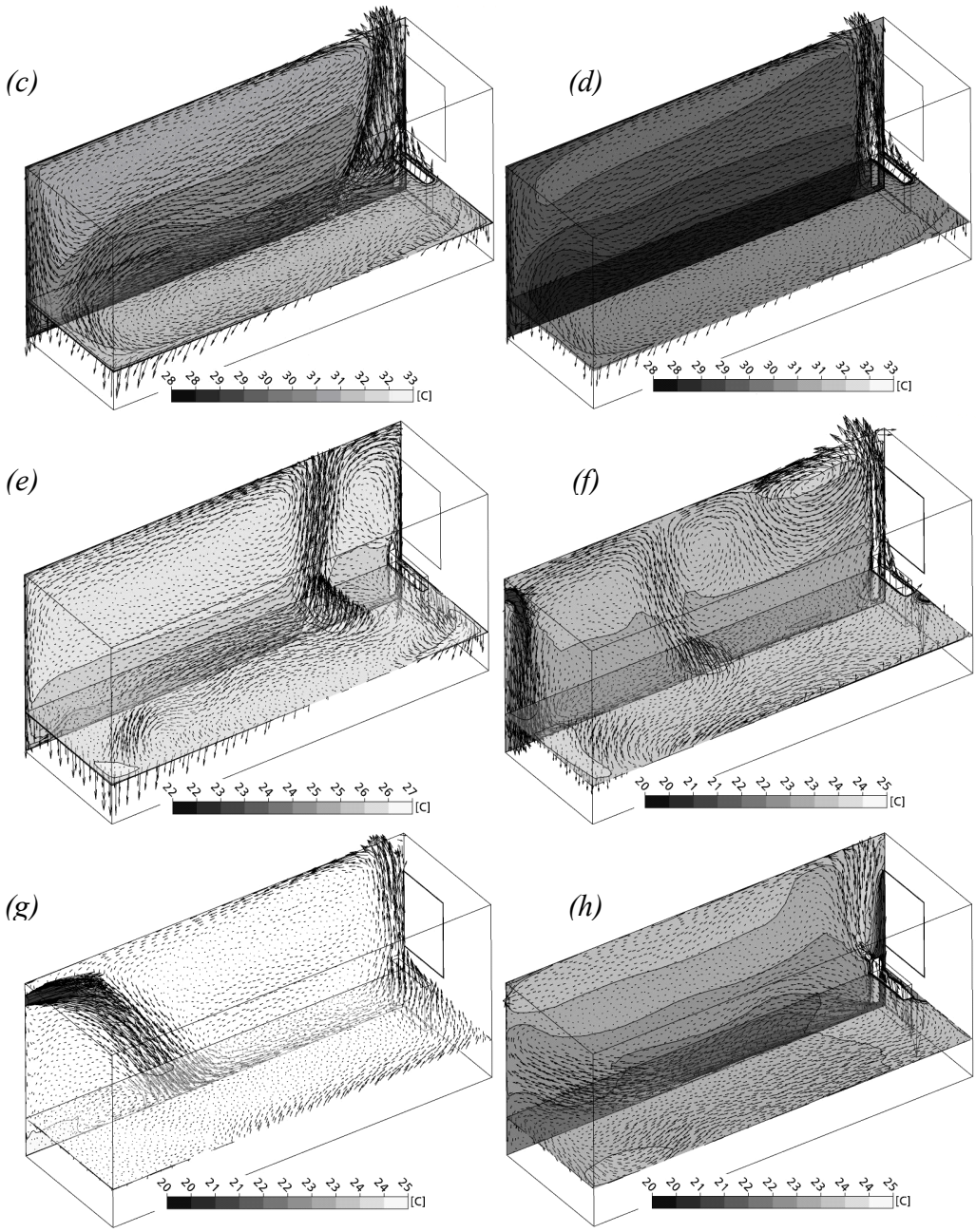


Fig. 4. Velocity vector field and temperature contours in middle vertical cross-section and 0.5m height horizontal cross-section for variants A (a),B1-B3 (b-d),C (e) and D1-D3 (f-h).

Significant uprising airflows are observed from the hot radiator (variants A, B1-B3 and D1-D3) and from the hot floor area (variant C), but downwards movement exists in all variants, the result of the lower temperature of the neighbouring rooms (20 °C) and the corridor (15 °C). If it is assumed that heat exchanges between the other rooms are excluded (adiabatic conditions are set), then the temperature distribution in the modelled room will noticeably increase, but with an airflow velocity in the room it will decrease.

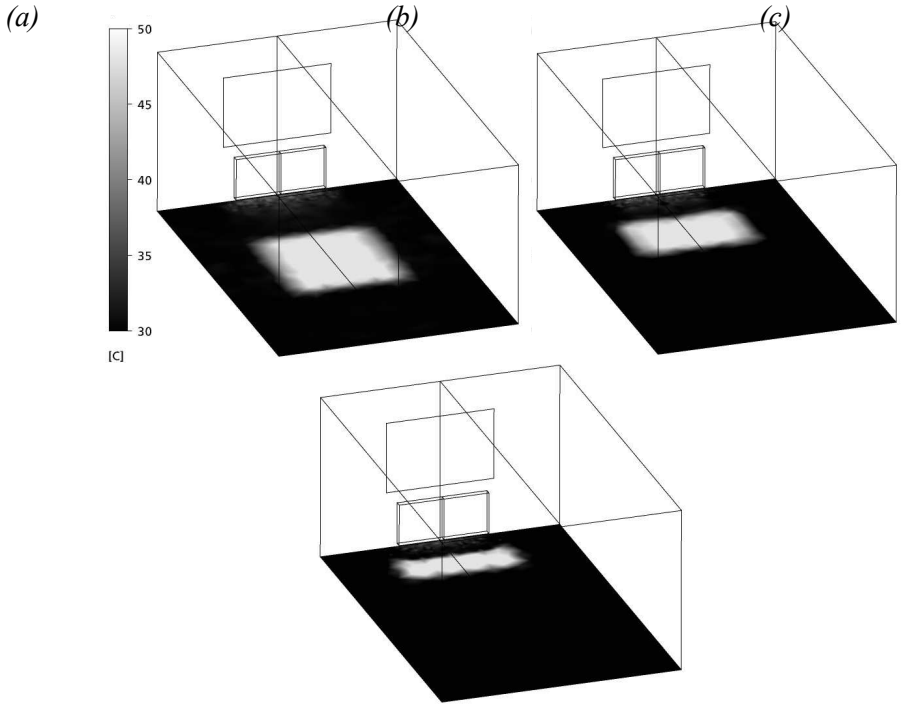


Fig. 5. Temperature distribution on the floor for variants B1 (a), B2 (b) and B3(c).

Significant uprising airflows are observed from the hot radiator (variants A, B1-B3 and D1-D3) and from the hot floor area (variant C), but downwards movement exists in all variants, the result of the lower temperature of the neighbouring rooms (20 °C) and the corridor (15 °C). If it is assumed that heat exchanges between the other rooms are excluded (adiabatic conditions are set), then the temperature distribution in the modelled room will noticeably increase, but with an airflow velocity in the room it will decrease.

Therefore we can assume that all considered variants with solar radiation can be improved to reach better thermal comfort conditions. It is possible to forecast that the best conditions for the modelled room can be reached as follows:

- the heater's power is being regulated in accordance with the solar radiation intensity and indoor temperature;
- a small overpressure is set in the room to exclude cold exterior air inflow (<0.3 *Pa*);
- temperature difference in the adjoining rooms is as small as possible;
- the angle of solar radiation attack is >45°.

If the above-mentioned conditions are fulfilled, intensive airflows will not be formed and an average temperature in the room will be acceptable for thermal comfort conditions.

Conclusion

A performed calculation for different variants of a living-room clearly demonstrates the influence of solar radiation sources and other factors on the heat balance and thermal comfort conditions in the room. The model allows estimation of temperature, airflow distribution and the tendencies of its changes, as well as a dependence on heat loss through the boundary structures under different solar radiation and ventilation conditions.

Obtained results show also the essential influence of some variable factors on the heat balance and the thermal comfort conditions. 3D modelling of a living-room including a solar heat source is very important for the correct representation of qualitative and quantitative heat transfer and convection processes in the living-room.

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Gendelis S., Jakovičs A., Dzīvojamās istabas ar Saules starojuma avotu un dažādiem robežnosacījumiem skaitliskā modelēšana

Lietojot programmatūru ANSYS CFX ir izveidots dzīvojamās telpas ar Saules radiācijas avotu 3D modelis, kas ļauj noteikt temperatūras un vidējoto turbulento plūsmu sadalījumu telpā. Fizikālo lauku sadalījumi un siltuma bilance tiek aprēķināta pie dažādām Saules starojuma avota jaudām un tās krišanas leņķa, kā arī dažādu spiedienu starpību starp atverēm telpas pretējās sienās. Kā termiskā komforta nosacījumi tiek analizēti gaisa kustības ātrumi, iekštelpu temperatūra un tās gradienti. Parādīts, ka solārais starojums būtiski ietekmē telpas siltuma bilanci un komforta apstākļus tajā.

Gendelis S., Jakovics A. Mathematical Modelling of a Living-Room with a Solar Radiation Source and Different Boundary Conditions

Temperature and average turbulent airflow distributions in the 3D model of a living-room with solar radiation modelling are considered using ANSYS CFX software. The distributions and heat balance are calculated according to the solar radiation source through the window and the pressure difference between openings in opposite walls. The airflow velocities and indoor temperature, including its gradients, are also analysed as parameters of thermal comfort conditions. It is shown that the solar radiation source has an essential influence on the heat balance of the room and on comfort conditions.

Гендель С., Якович А. Численное моделирование жилой комнаты с источником солнечного излучения и различными пограничными условиями

Используя пакет программ математического моделирования ANSYS CFX, создана 3D модель жилого помещения, включающая источник солнечного излучения, с помощью которой возможны расчеты распределения температуры и усредненных турбулентных потоков. Распределения физических полей, а также тепловой баланс рассчитываются при различных мощностях и углах падения солнечного излучения через окно, а также в зависимости от разницы давлений между отверстиями в противоположных стенах комнаты. Как характеристики теплового комфорта анализируются скорости перемещения воздушных масс, температура воздуха и ее градиенты. Показано, что солнечное излучение имеет важное влияние на тепловой баланс помещения, а также на тепловые условия комфорта.