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Magnetorheological Suspension Composed of Fiber Particles: Numerical Simulation of Anisotropic Behavior

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Abstract

This paper describes methodology of direct numeric calculation of fiber magneto-rheological suspensions at particle level. Magnetic forces along with bidirectional hydro-dynamic interactions are taken in account. The model does not include Brownian motion of particles. Both particle and fluid inertia are taken into account. An analytic hydrodynamic description for prolate spheroids is used, and the prolate spheroid-cylinder hydrodynamic equivalency condition is applied. The fluid flow is calculated using finite difference methods in vorticity-vector potential formulation. A home-made software was developed to perform these calculations. Test results show that the chosen approach is suitable for simulation of magnetorheological suspension composed of ferromagnetic fibers.

Introduction

Magnetorhological suspensions (MRS) is a kind of functional material, the rheological properties of which can be controlled my magnetic field. MRS is composed of small ferromagnetic particles suspended into a carrier fluid. In external magnetic field particles magnetize and interact one with another changing the attributes of MRS. Traditionally MRS components are spherical particles. It is expected that MRS with rod-like particles will be with different features and new applications.

1. General Information

Calculations are made in space of rectangular box $L_x \times L_y \times L_z$ (fig. 1). Coordinate system origin is placed in one corner, axis are directed along the edges, besides that *z*-axis is assumed to be the vertical axis, along which magnetic field is directed. The two horizontal sides are impenetrable walls, but four remaining vertical sides are periodic MRS boundaries. The lower horizontal wall remains motionless. The upper horizontal wall is moving in the direction of *x*-axis with velocity *U* (as shown in fig. 1) creating shear-flow.

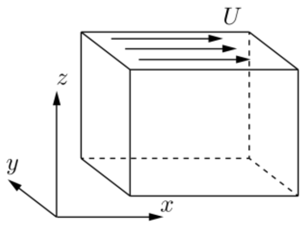

Fig. 1. Model space

MRS is simulated as two-phase fluid: discrete (Lagrange) phase consists of rod-like ferromagnetic particles, whereas continuous (Euler) phase is carrier fluid.

2. Discrete phase

The main algorithm is taken from [1]. Particle motion is simulated by calculating discrete versions of force balance equations

$$\frac{m}{\Delta t}\left(\vec{v}-\vec{v}_{n-1}\right)=\vec{F} \quad \text{and} \quad \dot{I}_{n-1}\cdot\vec{\omega}+\frac{1}{\Delta t}I\cdot\left(\vec{\omega}-\vec{\omega}_{n-1}\right)=\vec{T} \tag{1}$$

The index *n-1* means previous time step. *I* is particle inertia tensor, *m* is particle mass, $\vec{v}$ and $\vec{\omega}$ are velocity and angular velocity of the particle. The right sides of both equations are sum of all forces and force moments on the particles, and left sides are inertia forces. From these equations one can find particle velocities in current time step $\vec{v}$ and $\vec{\omega}$.

2.1. Hydrodynamic Forces

Hydrodynamic force equations are taken from analytic solution for single spheroid particle in laminar fluid flow [1, 2]. It can be done because of semi-empiric relation [3] which states the hydrodynamic equivalence of prolate spheroid and cylinder of same length (as shown in fig. 2) if $2b=\frac{1}{1.24}D\sqrt{\ln\frac{l}{D}}$, where $2b$ and D are the diameter of spheroid and cylinder respectively, and $2a = l$ are the length of both spheroid and cylinder.

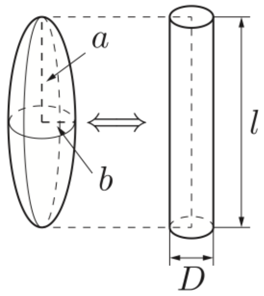

Fig. 2. Hydrodynamic equivalency of prolate spheroid and cylinder

2.2. Magnetic Forces

The description of magnetic forces is based on a simple "Magnetic Coulomb Force" model. Magnetic charges of opposite sign are placed at both ends of each rod; the magnetic force is given by

$$\vec{F}_{ij}^{m,*}=\frac{1}{3}\frac{3\mu_0}{4\pi}M_iM_jS_iS_j\frac{\left(\vec{z}_i\cdot\vec{e}_z\right)\left(\vec{z}_j\cdot\vec{e}_z\right)}{\left(r^*\right)^2}\frac{\vec{r}^*}{r^*},$$

where $\vec{r}^*=\vec{r}_i^*-\vec{r}_j^*$, $r^*=\left|\vec{r}^*\right|$ is distance between "magnetic charges", each of which belong to *i*-th and *j*-th particle, whereas S_i and S_j are particle cross-section areas respectively. The magnetic charge intensity depends on the angle between the rod and external magnetic field, therefore coefficients like $\left(\vec{z}\cdot\vec{e}_z\right)$ emerge. Total force on particle is sum of forces between charges.

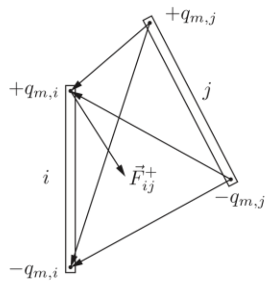

Fig. 3. "Magnetic Coulomb Force" model

2.3. Contact Forces

When two particles are completely or almost in contact, new interaction forces emerge: normal forces, friction forces and lubrication forces. Normal forces prevent particles from overlapping. Because the geometry of particles is not spherical, the direction of contact forces $\vec{n}_{ij}$ highly depends on particle alignment in the specific contact situation. For details on $\vec{n}_{ij}$ see [4]. Normal force is chosen to be compilation of exponential and linear functions:

$$\begin{cases} -F_c\exp\left(-\frac{d_{ij}+\delta_D}{d_0}\right)\vec{n}_{ij}, & \text{if } d_{ij}\le-\delta_D, \\ -F_c\exp\left(\frac{1}{2}-\frac{d_{ij}}{2\delta_D}\right)\vec{n}_{ij}, & \text{if } -\delta_D<d_{ij}\le\delta_D, \\ \vec{0}, & \text{if } d_{ij}>\delta_D, \end{cases}$$

where $d_{ij} = \left|\vec{g}_{ij}\right| - \frac{D_i + D_j}{2}$ is the distance between particle surface at the point of contact and δ_D is surface roughness. Vector $\vec{g}_{ij}$ connects points on both particle axis. In most cases (when $\vec{g}_{ij} \neq \vec{0}$) expression $\vec{g}_{ij} \parallel \vec{n}_{ij}$ is true. The rods overlap if d_{ij} is negative. Parameter d_0 controls sensitivity of the exponent, here is assumed $d_0 = D_i / 4$. Coefficient F_c is chosen to be comparable with typical system forces: viscous drag forces, dynamic drag forces and magnetic forces. Therefore F_c is chosen to be

$$F_c = \frac{3}{2}\pi\eta\bar{l}\,\overline{w}^* + \frac{1}{2}\rho\overline{D}\,\bar{l}\left(\overline{w}_\perp^*\right)^2 + \frac{1}{3}\frac{3\mu_0}{4\pi}\frac{\pi^2}{16}M^2\overline{D}^2,$$

where a bar over a quantity means the mean value over all particles.

For friction and lubrication forces, the relative velocity of particle contact points is needed. It can be defined using linear contact coordinates S_{ij} and S_{ji} with respect to particle centers along the axis of each particle:

$$\Delta\vec{v}_{ij}^* = \vec{v}_i - \vec{v}_{j,n-1} + S_{ij}\vec{\omega}_i \times \vec{z}_i - S_{ji}\vec{\omega}_{j,n-1} \times \vec{z}_j + \frac{1}{2}\left(\vec{\omega}_i + \vec{\omega}_{j,n-1}\right) \times \vec{g}_{ij},$$

where the approximation of unknown velocities $\vec{v}_j$ and $\vec{\omega}_j$ of *j*-th particle was taken from previous time step *n*-1.

Friction force acts in plane perpendicular to the normal force and is a function of its absolute value. The direction of friction force is defined by the projection of $\Delta\vec{v}_{ij}^*$ in the mentioned plane: $\Delta\vec{v}_{ij}^* - \left(\vec{n}_{ij} \cdot \Delta\vec{v}_{ij}^*\right)\vec{n}_{ij} = \left(\delta - \vec{n}_{ij}\vec{n}_{ij}\right)\Delta\vec{v}_{ij}^*$, where right hand side is rewritten using tensor notation. So the friction force can be written as

$$\vec{F}_{ij}^f = -\mu_f\left|\vec{F}_{ij}^n\right|\frac{\left(\delta - \vec{n}_{ij}\vec{n}_{ij}\right)\Delta\vec{v}_{ij}^*}{\left|\left(\delta - \vec{n}_{ij}\vec{n}_{ij}\right)\Delta\vec{v}_{ij}^*\right|},$$

where μ_f is the coefficient of friction. However $\Delta\vec{v}_{ij}^*$ cannot stand in denominator, because the velocities $\vec{v}_i$ and $\vec{\omega}_i$ of current time step should be extracted and moved from right hand side to left hand site in equation (1). Therefore as normalization coefficient will be used approximation $\overline{w}^*$, which stands for mean relative velocity between particle and fluid over all particles, so the friction force will be

$$\vec{F}_{ij}^f = -\mu_f\left|\vec{F}_{ij}^n\right|\frac{\left(\delta - \vec{n}_{ij}\vec{n}_{ij}\right)\Delta\vec{v}_{ij}^*}{\overline{w}^*}.$$

By taking into account surface roughness [5], lubrication force is given with expression

$$\vec{F}_{ij}^l = \vec{n}_{ij}F^l\left[D_i/2, D_j/2, \max\left(d_{ij}, \delta_D\right), \vec{n}_{ij} \cdot \Delta\vec{v}_{ij}^*, \Delta_{ij}\right],$$

Here

$$F^l\left(R_1, R_2, d, \dot{d}, L\right)=\begin{cases} F_\infty^l\left(R=R_{eq}\right), & \text{if } \left|F_\infty^l\right|<\left|L\frac{d}{dz}F_\parallel^l\right| \\ L\frac{d}{dz}F_\parallel^l\Big|_{R=R_{eq}}, & \text{in other cases,} \end{cases} \quad \text{and} \quad R_{eq}=2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1},$$

where

$$F_\infty^l=-\eta\frac{12\pi}{\sin\alpha}\frac{R^2}{d}\dot{d} \qquad \text{and}$$

$$\frac{d}{dz}F_\parallel^l=-\eta\dot{d}\left(A_0+A_1\frac{d}{R}\right)\left(\frac{d}{R}\right)^{-\frac{3}{2}},\ A_0=3\pi\sqrt{2}/8,\ A_1=207\pi\sqrt{2}/160$$

are taken from [6] and [7] respectively. $\Delta_{ij}\le\min\left(l_i, l_j\right)$ is particle overlap length in the direction of near parallel particle axis.

The sum of all contact forces on *i*-th particle is

$$\vec{F}_{ij}^c=\vec{F}_{ij}^n+\vec{F}_{ij}^f+\vec{F}_{ij}^l,$$

and in most cases this causes also force moment

$$\vec{T}_{ij}^c=\left(S_{ij}\vec{z}_i+\frac{1}{2}\vec{g}_{ij}\right)\times\vec{F}_{ij}^c.$$

2.4. Impulse Conservation

Because discrete and continuous phases are calculated consecutively and apart of each other, impulse conservation should be enforced specifically. This is done by ensuring hydro-dynamic force operation in both phases. For one hand force $\vec{F}_i^h$ is included in particle force balance equation (1) right hand side. For another hand reverse direction is provided by adding $-\vec{F}_i^h$ to fluid calculation equations shown below. Because fluid flow calculation equations does not contain force momentum, it should be converted to force pair as follows: $-\frac{1}{D_i}\vec{T}_i^h\times\vec{\tau}_i$ and $\frac{1}{D_i}\vec{T}_i^h\times\vec{\tau}_i$, which acts on $\vec{r}_i+\frac{D_i}{2}\vec{\tau}_i$ and $\vec{r}_i-\frac{D_i}{2}\vec{\tau}_i$ respectively, where $\vec{\tau}_i=\frac{\vec{T}_i^h\times\vec{z}_i}{\left|\vec{T}_i^h\times\vec{z}_i\right|}$.

2.5. Discrete Phase Boundary Conditions and Particle Positions

The boundary conditions of discrete phase say that horizontal walls should be impenetrable for particles. However vertical boundaries are cyclic in all other directions: each particle traversing some vertical boundary should reappear near opposite boundary in the direction of *x* or *y* axis. For fully cyclic calculations, eight virtual particles should be defined for each "real" particle in eight virtual spaces lying adjacent to the calculation space in *x* and *y* directions. Each virtual particle interacts with each "real" particle, but not with other virtual particles. Similarly is ensured the ferromagnetic properties of horizontal walls: particles are mirrored along the plane of each wall obtaining more virtual particles. By letting them interact with original particles, the ferromagnetic wall effect is obtained.

The goal of simulation is to calculate particle motion. Using above equations together with (1) particles velocities $\vec{v}$ and $\vec{\omega}$ in current time step can be calculated. And by knowing particle velocities, it is possible to calculate the new particle positions in current time step.

New particle position $\vec{r}$ with respect to the old $\vec{r}_{n-1}$ will be $\vec{r} = \vec{r}_{n-1} + \Delta t \cdot \vec{v}$, where Δt is discrete time step. Similarly new particle axis orientation will be $\vec{z} = \vec{z}_{n-1} + \frac{\vec{\omega}\times\vec{z}_{n-1}}{|\vec{\omega}\times\vec{z}_{n-1}|}|\vec{\omega}|\Delta t$.

3. Continuous Phase

Fluid flow are simulated by numerically calculating Navier-Stokes equations of fluid motion in vorticity-vectorpotential formulation [8]:

$$\frac{\partial\vec{\omega}}{\partial t} + \nabla\times[\vec{\omega}\times(\nabla\times\vec{\psi})] = \nu\Delta\vec{\omega} + \frac{1}{\rho}\nabla\times\vec{f},$$
$$-\Delta\vec{\psi} = \vec{\omega},$$

where $\vec{\psi}$ is vectorpotential such that fluid velocity $\vec{u} = \nabla\times\vec{\psi}$, $\vec{\omega} = \nabla\times\vec{u}$ is vorticity, ν is kinematic viscosity, ρ is density, and $\vec{f}$ are forces per volume. Given equation system is solved together with following boundary conditions on horizontal walls

$$\vec{n}\times\vec{\psi} = const, \quad \frac{\partial}{\partial\vec{n}}(\vec{\psi}\cdot\vec{n}) = 0,$$
$$\vec{n}\cdot\vec{\omega} = \vec{n}\cdot\nabla\times\vec{u}, \; \vec{n}\times\vec{u}_b = \vec{n}\times\nabla\times\vec{\psi}$$

and initial condition

$$\vec{\omega}_0(\vec{x}) = \nabla\times\vec{u}_0(\vec{x}).$$

The first boundary conditions allow for vectorpotential components ψ_x and ψ_y on both walls to be defined as a free constant, therefore employing common reasoning, one can find that

$$\psi_x\big|_{z=L_z} = \psi_x\big|_{z=0} = 0, \quad \psi_y\big|_{z=0} = 0, \quad \psi_y\big|_{z=L_z} = -\frac{UL_z}{2}.$$

However by the same reasoning, on vertical side boundaries vectorpotential should have cyclic condition ($\vec{\psi}$ values coincides on opposite sides), which enforces periodic continuous phase.

Given fluid flow equations are discretized using difference scheme and are calculated on mesh nodes. Calculated values on nodes with tri-linear interpolation are used to calculate needed quantities anywhere in calculation space. Volume forces are similarly distributed to mesh nodes by using similar tri-linear interpolation: $\vec{F}_i = \vec{F}(\vec{r})N_i(\vec{r}-\vec{r}_0)$, where i stands for local mesh node index and N_i is respective node tri-linear interpolation function.

4. Results

Single particle in shear-flow performs rotation along its center of mass. Such rotations are called Jeffery orbits and are theoretically derived. In numeric simulations obtained rotation periods are with good agreement with theoretical values with max 5% error. Theoretical Jeffery orbit period [9] is $T = \frac{2\pi}{\dot{\gamma}_0}\left(r_e + \frac{1}{r_e}\right)$, where $r_e = \frac{1.24 r_c}{\sqrt{\ln r_c}}$ [3] and $r_c = l/D$. Quantity $\dot{\gamma}_0 = \frac{U}{L_z}$ is shear flow gradient.

Performing simulations with small number of particles, one can observe, that fibers try to make vertical chains as a result of magnetic force interactions between particles. Similar chains can be observed when performing simulations with greater volume fractions (as seen in fig. 4).

Conclusions

The test results confirm that the numerical algorithm, and software developed are adapted to simulate the magnetorheological suspension in the case of the rod-like ferromagnetic particles. Momentum exchange between the two phases ensures that the quantitative estimate of the effective viscosity is possible if the relatively large amount of particles is simulated.

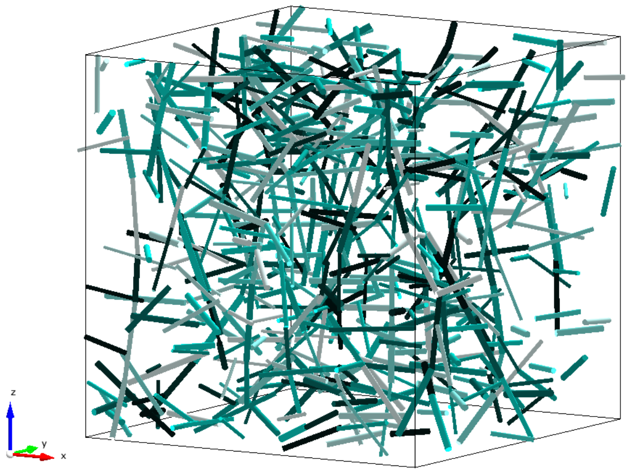

Fig. 4. Dilute fiber solution, volume fraction $\phi \approx 0.5\%$

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