Suppression of Temperature Fluctuations by Rotating Magnetic Field in a Large Scale Rayleigh-Bénard Cell

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Abstract

Transition between flow regimes is studied experimentally in a cylinder of liquid mercury heated from below under action of a rotating magnetic field. The latter creates a rotating flow which efficiently suppresses the temperature fluctuation caused by unstable thermal stratification. Our experiment confirms that this effect persists to Grashof number as high as about 10^9. An intermediate range is observed for $Gr > 2 \times 10^8$ with the temperature fluctuation suppressed in the core but much less near the side wall.

Introduction

The yield of the Czochralski crystal growth process can be increased by rising the crystal-to-crucible diameter ratio. Such approach, however, requires also a higher initial filling level. Otherwise the crystal would be unpractically short. A higher filling level, in turn, quickly destabilizes the flow as the governing control parameter is proportional to the layer depth to power four [1]. The melt is stabilized by mechanical rotation of the crucible in the conventional Czochralski growth. The melt rotation can be also driven magnetically and the rotating magnetic field (RMF) is long known [2-4] to suppress the buoyancy instability. It turned out [5], that an RMF driven flow serves this purpose at a much lower rotation rate than the mechanical rotation of crucible. Besides, the use of RMF instead of crucible rotation may simplify the construction of the puller.

The temperature field stabilization by RMF has been studied up to now in relatively small volumes with a maximum Grashof number of about 6×10^7 [5]. The following considerations suggest that the effect may modify as the cell volume is further increased. The maximum azimuthal velocity of the RMF driven flow is determined by inertial forces in the core provided that the dimensionless value of the magnetic force does not exceed certain threshold. This is observed as the `3/2' scaling [6] in the function of angular velocity vs. the magnetic force. As the volume of cell increases the turbulent friction becomes the ruling retarding force causing a shift to a `1/2' angular velocity scaling [5-7]. Since there is a qualitative change in the flow there may also be a qualitative change in its stabilizing properties. Main goal of our study is to clarify this question. For this purpose we repeat the previous experiment [5] with mercury instead of gallium as the working liquid.

The high density of mercury provides a very small kinematic viscosity and, thus, a high Grashof number. The combined effect of lower kinematic viscosity and higher thermal expansion coefficient yields a factor of about 20 (relative to gallium) for the Grashof number in the cell of fixed geometry and temperature difference. Mercury has an advantage also in terms of the required heating/cooling power to maintain the temperature gradient since its heat conductivity is remarkably low. The Prandtl number of mercury 0.025 does not differ considerably from that of gallium and matches well conditions in main semiconductor melts.

1. Experiment Description

The cell is made of a plexiglass tube with inner diameter $2R_o = 170$ mm closed by copper discs from both ends so that the distance between them is 170 mm. Each disk is 30 mm thick and has two circular channels for heating/cooling agent. The cell is filled with mercury. The surface of end-discs in contact with mercury is coated with 0.1 mm thick layer of aluminium oxide for the purpose of electric insulation. Two thermostats control the temperature of the covers. The thermostat of the upper cover is equipped with a chiller providing about 500 W cooling power at -10°C.

The following properties of mercury are used to calculate dimensionless critera: density $\rho = 1.36 \times 10^4$ kg/m^3, kinematic viscosity $\nu = 1.2 \times 10^{-7}$ m^2/s, electric conductivity $\sigma = 1.04 \times 10^6$ S/m, thermal expansion coefficient $\beta = 1.8 \times 10^{-4}$.

Temperature is measured with miniature PT1000 thermistors of dimensions $2 \times 2.3 \times 0.6$ mm. Three thermistors are arranged regularly around the circumference of the cell's side wall on its mid-plane. A fourth thermistor is placed on the axis 17 mm below the top cover. Two more thermistors are placed in the centre of both copper end walls in immediate vicinity of the inner surface. The readings of sensors are converted by *Data Translation* DT9806 data acquisition card and registered on a computer memory at 12.5 Hz frequency. For a given temperature difference between thermostats a set of measurements is made with different (fixed) RMF. Duration of each run is either 15 min for RMF induction below 2.5 mT and 6 min otherwise. The maximum applied temperature difference ΔT between covers is 22.3°C that corresponds to a Grashof number

$$Gr = \frac{\beta g T' R_o^4}{\nu^2}$$

as high as 0.84×10^9. The acceleration of free fall is denoted by g and $T' = \Delta T / 2R_o$ is the imposed temperature gradient.

The RMF inductor is made of 6 equal coils in form of rounded rectangles 22 cm wide and 55 cm high. Each coil has 200 winding of 0.95 mm diameter copper wire. The coils are arranged in a regular hexagon. A ferromagnetic yoke is made of 80 winding of 3.5 mm diameter steel wire wound around the coils. He inductor is fed by a variable amplitude three phase alternating current of 50 Hz frequency. The maximum RMF flux density (root mean square value) is $B_o = 4$ mT and the field non-uniformity is below 10%. This corresponds to a maximum magnetic Taylor number

$$Ta = \frac{\sigma \omega B_0^2 R_o^4}{\rho \nu^2}$$

of about 1.5×10^9, where ω is the angular frequency of the RMF. The shielding factor or the dimensionless frequency $S = \sigma \mu_0 \omega R_o^2 = 3$ is low enough to neglect the skin-effect (μ_0 is the vacuum magnetic permeability).

2. Results

A weakly correlated un-periodic temperature variation was observed at the side wall in absence of RMF. As the RMF was imposed the fluctuation amplitude initially grew slightly and became increasingly periodic with a clear signature of a rotating wave (Fig. 1a). After reaching a certain threshold value of the RMF strength the fluctuation amplitude fell sharply (Fig. 1b) becoming simultaneously irregular. The effect of fluctuation suppression was much more pronounced in the core flow (Fig. 1 cd). The dependence of the threshold value of the RMF strength that damps fluctuation vs. the Grashof number is one of the main goals of the study. To find it out we took the dependency of the fluctuation amplitude characterized by

standard deviation δT on Ta for different Gr. Fig. 2 shows the scaled standard deviation $\delta T/\Delta T$ for the signal measured in the core (a) and near the side wall (b). The fluctuation was more suppressed in the core and followed approximately $O(Ta^{-2})$ scaling. The differences between curves in Fig. 2(a) and Fig. 2(b) makes it difficult to formulate a single criterion of the threshold value Ta_c for both places of measurements. Therefore we defined several thresholds:

- Ta_0: dimensionless fluctuation amplitude $\delta T/\Delta T = 0.01$ in the core (Fig. 2a);
- Ta_1: $\delta T/\Delta T = 0.05$ at the side wall (Fig. 2b);
- Ta_2: the lower "knee-point" — the maximum of 2nd order derivative of the δT (Ta) function (see Fig. 2b)

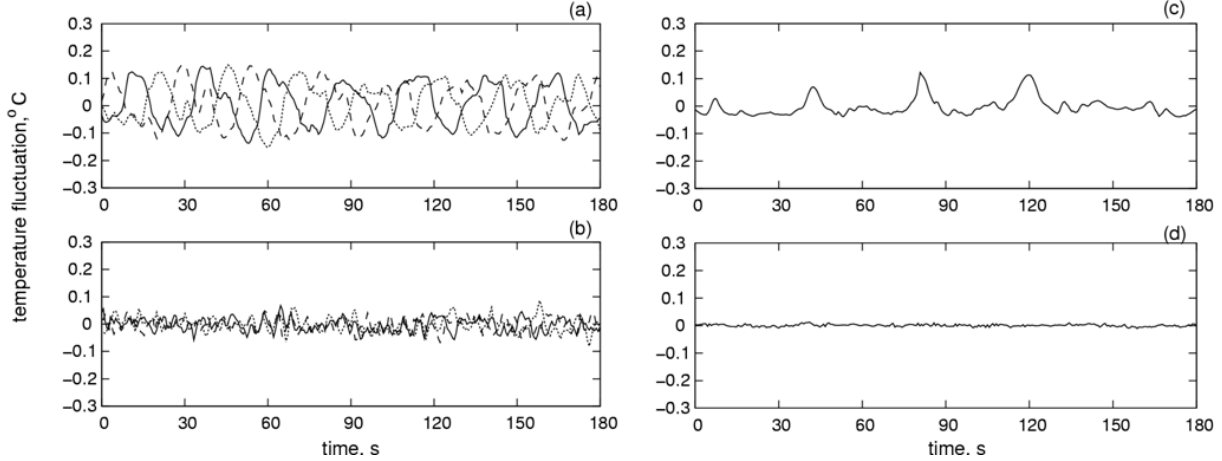and plotted them against the Grashof number in Fig. 3.

Fig. 1. Temperature fluctuation near the side wall (a, b) and in the core (c, d) in two regimes with $\Delta T = 1.15^\circ$C ($Gr = 4.3\times10^7$) and $B_o = 0.44$ mT (a, c) or 0.61 mT (b, d) ($Ta = 1.70\times10^7$ or 3.29×10^7, respectively). Different line type in (a, b) distinguishes signals from three sensors at the side wall.
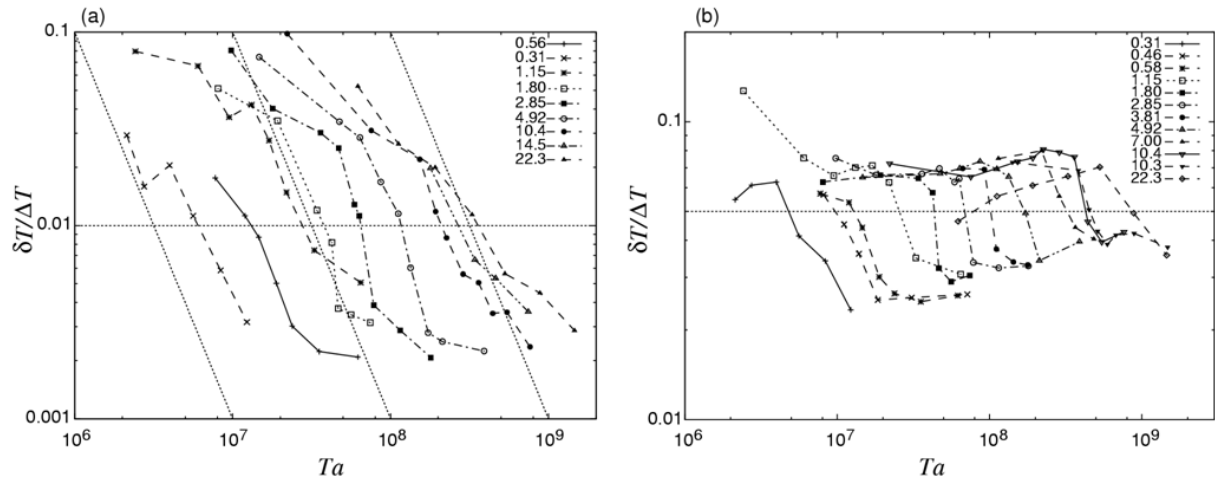

Fig. 2. Scaled standard deviation of temperature fluctuation vs. Ta for different ΔT in the core (a) and at the side wall (b). Dotted lines in (a) correspond to Ta^{-2} slope.

The regime change can be observed not only in the fluctuation amplitude but even better in the temporal structure of the signal, particularly for high Gr. Fig. 4 depicts transition in terms of the characteristic value of autocorrelation defined as its average absolute value over shift times from 0 to 60 s. It is observed that this quantity varies with Ta more sharply than the fluctuation amplitude. The corresponding threshold value Ta_3 (intersection with dotted horizontal line in Fig. 4) is shown also in Fig. 3 where it matches well Ta_2.
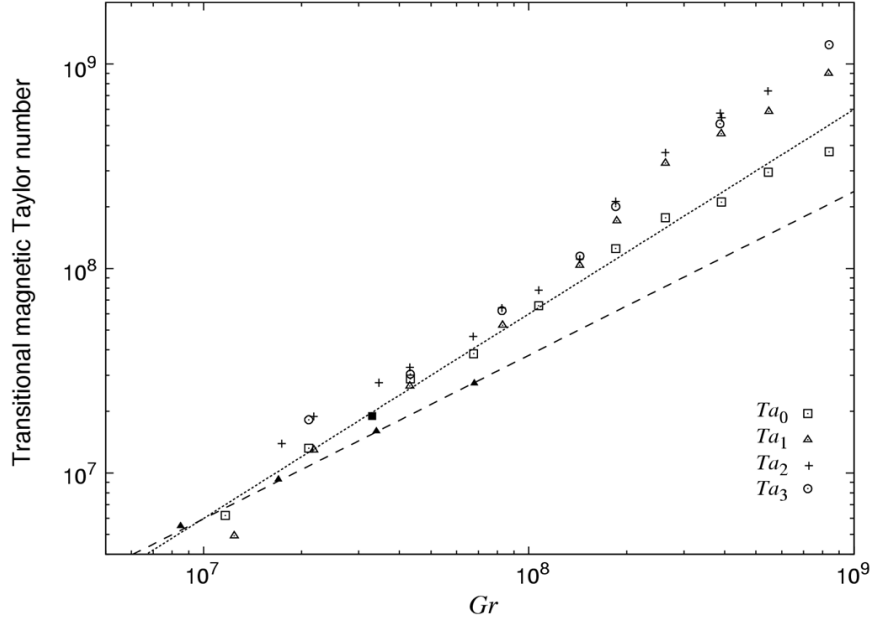
Fig. 3. Transitional magnetic Taylor number vs. Gr. Dashed and dotted lines depict $15Gr^{0.8}$ and $0.6Gr$, respectively. Results of [5] (Fig. 4) are shown with filled triangles. Filled box is evaluated as Ta_0 from Fig. 5(a) in [5]
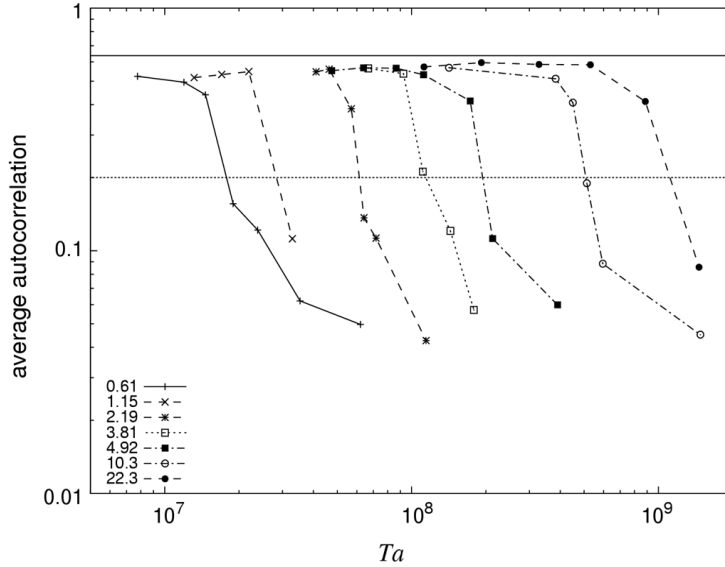

Fig. 4. Characteristic value of signal autocorrelation near the side wall vs. Ta at different ΔT as identified in the legend. Solid horizontal line shows theoretical limit of a harmonic function

The angular velocity of the flow was measured by the time shift maximizing the correlation between two azimuthally displaced probes on the side wall. Note that the method works also with a suppressed thermal wave. In that case the maximum value of the correlation decreases, but the maximum remains pronounced (Fig. 5). The angular velocity expresses as $\Omega = \Delta\varphi/\tau_{max}$, where τ_{max} is the time shift at which the maximum correlation is observed for probes displaced by the angle $\Delta\varphi$. The results in terms of the Reynolds number $Re = \Omega R_o^2/\nu$ in Fig. 6 agreed well with similar measurements in a smaller cylinder [5].

3. Discussion and Conclusions

The flow regime change caused by an RMF in a cylindrical Rayleigh-Bénard cell with liquid mercury is studied experimentally. This study continues our earlier work on this phenomenon [5,8] in a range of a higher Grashof numbers reaching values as high as 0.84×10^9. The results show that the temperature fluctuation caused by unstable thermal stratification can be significantly suppressed in this range. The standard deviation of temperature fluctuation measured at the distance 1/10 of the cell's height on the axis is suppressed below one per cent of the imposed temperature difference ΔT when the magnetic Taylor number reaches a value of about $0.6Gr$. The temperature fluctuation at the mid-height of the cell's side wall is suppressed less than in the core to a value of standard deviation 2 to 4 per cents of the imposed temperature difference. The effect of RMF on fluctuation amplitude decreases for higher Gr near the side wall but remains little changed in the core.
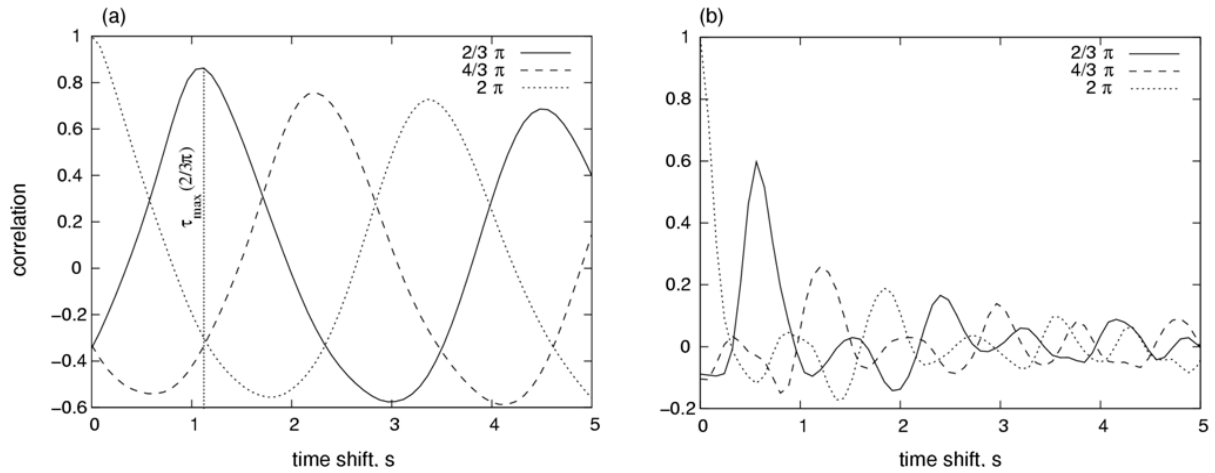

Fig. 5. Two-point correlations for differently displaced probes with $\Delta\varphi$ as given in the legend; $\Delta T = 10.3^{\circ}$C ($Gr = 3.9 \times 10^8$); $B_o = 2.3$ and 4.1 mT ($Ta = 4.5 \times 10^8$ and 1.5×10^9) for (a) and (b), respectively

An intermediate range forms at $Gr > 2 \times 10^8$ with the rotating thermal wave suppressed in the core but persistent at the side wall. Such regime is observed for Ta in the range of [0.6 Gr : 1.2 Gr]. The current results do not give any information on the spatial extent of the thermal wave. This intermediate regime may be caused by a change of the dominant retarding force. Up to $Ta = 3 \times 10^7$ the angular velocity follows a $Re \sim Ta^{2/3}$ scaling [5] which is established by the Coriolis force in the core [6]. This scaling is replaced by $Re \sim Ta^{1/2}$ [5,7] for higher Ta and may be explained [6] by the action of turbulent friction. The latter is localized near the side wall. As a consequence, some radial flow structure arises with the outer shell dominated by turbulent friction and an inner part dominated by inertia [6]. Likely, this outer shell does not suppress the thermal wave as much as the inner core. If this explanation is correct then the rotating thermal wave region should be relatively thin.
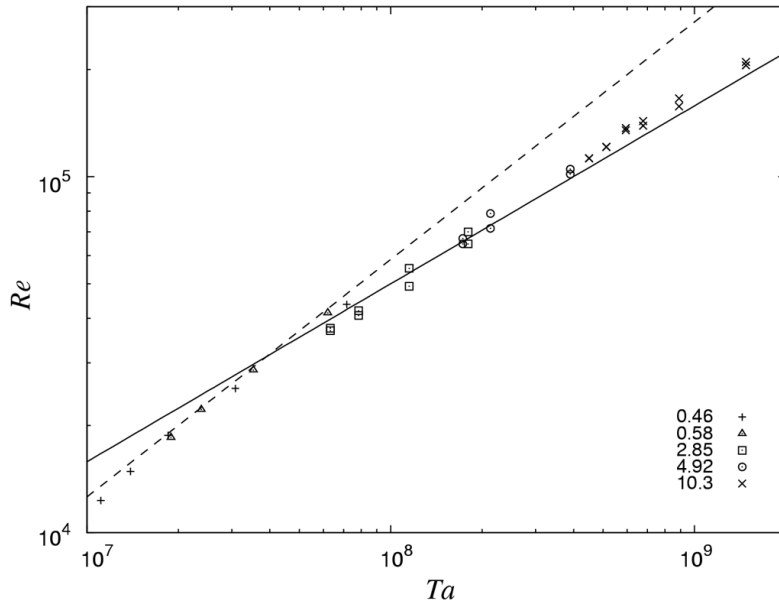
Fig. 6. Angular velocity vs. magnetic force measured at different ΔT as given in the legend. Dashed and solid lines show relations $Re = 5Ta^{1/2}$ and $Re = 0.27Ta^{2/3}$, respectively

The range of the considered Gr partially overlaps with that of the previous study [5]. The now observed Ta_c exceeds the previous results by some 20 to 40% for $Gr < 7 \times 10^7$. The reason for that may be a different value of the dimensionless frequency of the RMF. The value of S=7.5 was close to the optimum for fluctuation suppression at the wall in [5]. In the current experiment the power supply had a fixed frequency of 50 Hz that corresponds to $S = 3$. Note that the critical value in the core (filled box in Fig. 3) for $S \approx 3$ evaluated from Fig. 5(a) in [5] matches the current results (Ta_0) very well. The measured angular velocity of the thermal field also matched well the previous observations. For Ta in $[2 \times 10^7 : 10^9]$ this velocity was $Re = 5\, Ta^{1/2}$. That agrees in 10% tolerance to the results in Fig. 8 of [5].

The current results confirm the conclusion [5] that an RMF is a much more efficient means of stabilization of liquid metal heated from below than the mechanical rotation for Gr as high as 10^9. The advantage of RMF in terms of the stabilizing angular velocity remains overwhelming in this range.

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