Numerical Study of Magnet Systems for Lorentz Force Velocimetry in Electrically Low Conducting Fluids

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Abstract

This paper describes the first steps in the design process of a magnetic system for a new Lorentz force velocimeter (LFV) prototype in electrically low conducting fluids like glass melts. Therefore the network modeling method was used to choose the general magnetic source type. Special assemblies of different magnet system principles were compared using the 3D Simulation tools MAXWELL® and COMSOL®. These simulation methods were validated on a predefined test setup before. Finally the most promising rough draft was optimized geometrically using parametric sweeps in the 3D Simulation tools.

1. Introduction, Background and Theory

To measure the flow velocity in glass melts, which are low conducting and high viscous fluids, means to detect velocities in the region down to $v = 1 \text{ mm/s}$ of fluids at a temperature of more than $T \approx 1100 \text{ °C}$ with characteristic conductivities of only $\sigma \approx 10 \text{ S/m}$. There is no flow measurement technique existing, which is working well under these conditions. For such applications the Lorentz force velocimetry in principle is excellent suited, because of its non-contacting character. For typical glass melt applications with volumes of $V = 0.1 \text{ dm}^3$ in a magnetic field of $B = 0.1 \text{ T}$ the expected Lorentz forces are in the tiny region of only $10^{-6} - 10^{-7} \text{ N}$. These forces are hard to measure and so one idea to increase them is to use much stronger magnet systems. The investigations on suitable magnet systems for LFV and their design is the topic of this paper.

The Lorentz force velocimetry theory is based on the magneto-hydrodynamic (MHD) interaction of moving fluids with magnetic fields. This interaction is known since Michael Faraday attempted to determine the voltage induced in the Thames by the magnetic field of the earth in the beginning of the 19th century [2]. The most common commercial application for this kind of interaction between moving fluid and applied magnetic field is the so called inductive flowmeter. The theory of this MHD effect was well described by J.A. Shercliff in 1963 [6]. Inductive flowmeters are widely used for the measurement of flow velocities in cold electroconductive fluids with conductivities $\sigma > 50 \text{ mS/m}$ like beverages, wastewater and chemicals. But with this technique it is only possible to measure flow velocities at moderate temperatures up to 200 °C, because some electrodes have to be putted into the fluid to measure the induced voltages. These electrodes would get corroded too fast in high temperature and aggressive fluids like glass or metal melts.

The Lorentz force flowmeter goes one step further. Here the induced voltages aren’t measured directly, but the forces which arise if the applied magnetic field interacts with these induced electric currents (see Fig. 1). In principle eddy currents are directed to oppose their reason – here the movement through the magnetic field –. So they generate a Lorentz force which decelerates the movement of the fluid. According to Newton’s law (actio=reactio) the same force acts on the magnetic field source and moves it in flow direction. Measuring this
force gives a flow velocity dependent signal. So no direct contact with the fluid is necessary anymore and it becomes possible to measure also hot fluids like metal and glass melts.

![Diagram of flux density isolines](image)

**Fig. 1.** Solid electroconductive bar moving through two permanent magnets (left: theoretical principle, right: mapped current vectors in the center shape)

The detailed theory for Lorentz force flowmeter was described by Prof A. Thess in 2007 for the first time [8]. Out of this theory the approximation formula for the Lorentz force density can be derived:

\[ f \sim \sigma v B^2. \]  

(1.1)

This approximation shows some fundamental conclusions. First, the conductivity \( \sigma \) and the flow velocity \( v \) given by the specific fluid application have a linear influence on the resulting force. Second, the magnetic flux density \( B \) has a quadratic influence on the final force and gives therefore the basis for the investigations in this paper.

While some first experimental measurements on real metal melts were done in the last years [5, 7], there is no existing prototype for LFV on electrically low conducting fluids like electrolytes or glass melts. The challenge of LFV in electrolytes is the much smaller generated force due to their smaller conductivities (see also equation 1.1). To prove the suitability of the LFV technique on electrolytes under laboratory conditions is the goal of one research area of the research training group “Lorentz force velocimetry and eddy current testing” funded by Deutsche Forschungsgemeinschaft. Here a first prototype test channel will be build up, which needs sophisticated force measurement and magnet systems to perform LFV in the same way, like it was done on metal fluids before. Due to the fact that the expected resolution of the planned force measurement system is \( 10^{-7} \) N, the generated Lorentz force must be higher than \( 10^{-5} \) N. Furthermore the chosen force measurement system works like a kind of pendulum and so it initially limits the maximum weight of the magnet system to 1 kg overall mass. In this first period of sponsorship the magnet systems should be assembled out of classical magnet sources like copper coils or permanent magnets. Afterwards high temperature superconductors are planned to be used.

2. General Magnetic Source Consideration

One important question to answer is, whether an electrically or a permanent excited magnet system is better suited for this application. With coils it is easily possible to change the magnetic flux density directly to modulate the Lorentz force. This could be useful for investigating the direct dependency between the strength of the magnetic field in the test
channel and the resulting Lorentz force. However, an additional power supply and a cooling unit are needed to reach sufficient current densities in the coil. Both need a direct contact to the coil, but must not affect the coil with a force that is higher than the resolution ($10^{-7}$ N) of the force measurement technique. Furthermore all electrically excited magnetic systems have an active temperature drift due to the electrical losses in the coil. This drift would additionally modify the measured force and must be compensated too.

A permanent magnet under laboratory conditions instead needs no electrical contacts, no cooling and has a well-defined passive temperature drift depending on the room temperature only. In applications of the LFV it is basically not necessary to change the flux density. Beneath the mentioned advantages of permanent magnets, there is another beating argument against the usage of an electrically excited system. A coil without any cooling is much heavier than a permanent magnet, which produces the same resulting magnetic flux.

This becomes obvious if one compares two very simple setups with the help of network models. In one a coil and in the other a permanent magnet drives the magnetic flux through a perfect magnetic conducting iron yoke to the air gap, where the flow channel goes through (see Fig. 2).

![Network elements](image)

Fig. 2. Network elements in an electrically excited magnet system (left) and a permanent excited magnetic system (right)

It is assumed that the copper coil has a maximal current density of $j_{zul} = 10$ A/mm² and a copper fill factor of $k_{cu} = 0.8$. This coil is compared to a permanent magnet of neodymium iron boron of grade N48 operating in the energy maximum. The flux density, which both systems should generate, amounts $B_\delta = 0.2$ T in the air gap with the cross section of $A_\delta = 50 \times 50$ mm² and a length of $l_\delta = 52$ mm. The stray field losses in the system are assumed to $\sigma = 0.5$ and the flux density in the iron yoke should be $B_{yoke} = 2$ T. Out of these conditions one can easily derive the equations for the mass of the both magnetic sources. So the coil of the electrically exited system becomes around $m_{\text{coil}} \approx 3.1$ kg while the permanent magnet producing the same magnetic flux in the air gap becomes only $m_{\text{pm}} \approx 400$ g. This means, that for static applications of magnet systems permanent magnets are much more efficient than electrically excited coils, as long as uncooled copper coils are in focus. Out of this obvious result a permanent magnet system must be used in the first prototype.

3. FEM – Validation for MAXWELL® and COMSOL®

Because sophisticated magnet systems mostly are hard to transform into simplified magnetic networks like shown in Fig. 2, it is necessary to perform full 3D simulations of them instead. Therefore it is necessary to validate the used 3D simulation tools on a well-known existing setup to prove their correctness before. On a prototype magnet system for Lorentz force velocimetry on metal fluids some calibration measurements were done in June 2009. First the magnetic field strength in this magnet system was fully described using a 3D pattern measurement. Then a solid aluminum bar was moved well-defined through the magnet system
and the resulting Lorentz forces were measured for different moving velocities \((v = 0.03 \text{ m/s} - 0.06 \text{ m/s})\). Both parts of this calibration procedure were investigated using MAXWELL® 3D transient and COMSOL® 3D static simulations. Both simulations are in very good agreement to the measurement results of the forces (see Fig. 3).

Fig. 3. Force vs. velocity diagram for dry-calibration measurement and comparing simulations in MAXWELL® and COMSOL® of an experimental LFV for metal melts

The differences between the simulations and the measurements could be caused by deviations of the assumed material properties from the unknown real ones (conductivity of the aluminum bar, B-H curve of the iron). The difference between the simulation results can only be caused by the different meshes and the different solution types. Nevertheless the agreements are so good (<10%), that these simulation tools can be reliably used to simulate the expectable forces in further studies of more complicated magnet systems for LFV. These simulations at the moment only work with solid bodies. In the case of low conductivities the magnetic Reynolds number and the Hartmann number are very low, which means that the applied magnetic field is not deformed by the moving electrolyte and the fluid profile of the b [1]. Therefore a uniform fluid profile can be assumed, which finally behaves nearly like a solid body with the same conductivity like an electrolyte. In the further investigations the design process of the magnet system is based on the approximation of the fluid as such a “solid body”. The real dependency of the Lorentz force on the fluid profile is investigated in another part of the Research training group as well.

4. Evaluation of Special Magnet Designs

One interesting idea for the magnet system design is to split it into two parts, whereas one element is fixed on the ground and only the rest is interacting with the fluid. So this splitting allows reducing the total mass, which finally hangs on the force measurement system, to only that part, which is directly interacting with the fluid. For example in electrically excited magnetic systems the coil could be fixed on the ground and the iron yoke, interacting with the fluid, goes through it with a small air gap between the yoke and the inner wall of the coil. This air gap is essential to detect a tiny deflection of the yoke caused by the Lorentz force. Another idea is to put the iron yoke on the ground and move some permanent magnets relative to it with a small air gap between the magnets and the yoke.

The center position of the iron yoke in a coil cavity with a tiny air gap between them will be unstable. Here the reluctance force pulls the iron to one wall of the inner coil cavity. Similarly, the permanent magnets moving relative to the fixed iron yoke will be affected by this force as long as they are not in an absolute central position. This reluctance force is much higher than the acting Lorentz force for realistic yoke dimensions. Due to this it is physically not possible to split the magnetic system. This means all parts of the magnetic system – source and yoke – have to be assembled as one closed magnetic circuit and have to be hanged on the force measurement system completely.
Some other magnetic designs, which seem to be suitable for this application, are designs where the magnetic field is not acting perpendicular to the fluid flow, but parallel. In Fig. 4 left a magnetic rectangle of permanent magnetic material is assembled around the channel. This design generates moderate Lorentz forces too, but the forces of a classical permanent magnet system out of two magnets – one on each side –, producing a flux perpendicular to the channel (see Fig. 1) are higher.

![Diagram](image)

Fig. 4. Cut through an parallel magnet system (left) and Halbach rectangle with mapped magnetization directions (right)

All in all, there is only one design, which seems to be better suited for LFV at the moment. It is the so called Halbach-cylinder, but not in the usual construction [4] but in a modified design for rectangular cavities like described in [3]. Without going too deep into detail, also the promising Halbach design is not suitable at the moment. An optimized design with 1 Kg overall mass would result in triangular segments with very small edge angles (see Fig. 4 right). These segments are very fragile due to the brittle behavior of neodymium iron boron and can therefore not be assembled in such a structure. Furthermore the limitation in the total mass results in a very thin Halbach design. This leads to high stray fields, which makes the structure ineffective and so the generated forces are not significant higher than in the magnet designs mentioned before. Finally the simplest design of only two permanent magnets seems to be the best choice for the planned prototype (Fig. 1).

5. Parameterization

After choosing the general rough draft of the prototype, the geometry of the magnets has to be investigated in detail. The numerical simulations show that the generated Lorentz force depends strongly nonlinear on each length scale – length \( l \) (in direction of the flow), height \( h \) (perpendicular to the ground) and thickness \( d \) (perpendicular to the channel and parallel to the ground) – of the permanent magnets (Fig. 5 left). To express these nonlinearities in one formula seems to be impossible and so a parameter study was done to prepare an overview about the general behavior of the force depending on the mass of used magnetic material. By parameterizing the length \( l \) and the thickness of the permanent magnets a set of diagrams similar to Fig. 5 right can be obtained. Here the force behavior for a fixed thickness \( (d = 25 \text{ mm}) \) and varied lengths \( (l = 20 \ldots 60 \text{ mm}) \) and masses \( (m = 400 \ldots 800 \text{ g}) \) are shown. Comparing these diagrams for different thicknesses \( d \) gives an optimized geometry, which generates as much as possible Lorentz force with a predefined mass of magnetic material.
Fig. 5. Quantitative dependencies of the Lorentz force on length $l$, height $h$ and thickness $d$ (left) and the parameterization result for one fixed thickness and different masses of magnetic material (right) for a fluid with the cross section $A_{FL} = 50 \times 50\, \text{mm}^2$, the conductivity $\sigma = 4\, \SI{S}{\text{m}}$, the flow velocity of $v = 5\, \SI{m}{/s}$, channels walls of $d_{W} = 7\, \text{mm}$ and an additional air gap of $\delta_{air} = 1\, \text{mm}$

Conclusions and Outlook

The first prototype for the planned channel will be a set of two permanent magnets, magnetized in the same direction and located on both sides of the channel, so that the magnetic flux goes perpendicular through the channel like in Fig. 1. These magnets will be connected with glass or carbon fiber composites to keep the total mass within the limit of 1 kg. A classical coil system instead is ineffective and so not interesting for this application. If the exact dimensions of the channel are known, the magnet geometries can be easily optimized by using a parameterization like described in section 0. The direct optimization is hard to realize due to the strong nonlinear behavior of the Lorentz force depending on the geometries of these magnets. Once the magnet system is build up the resulting magnetic field will be characterized using a 3D pattern measurement. Then first LFV measurements can be done to validate the simulations. Afterwards a more sophisticated Halbach design is planned to increase the force even more. This makes higher masses for the magnet systems necessary, but the generated Lorentz force can also be increased by more than one order of magnitude.

References


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