

Eddy Current Interaction of a Magnetic Dipole With a Translating Solid Bar

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Abstract

The drag force acting on a magnetic dipole due to the motion of a conducting rectangular bar in its field is computed by finite element analyses for different orientations of the dipole. The study is motivated by the novel techniques termed Lorentz Force Velocimetry (LFV) and Lorentz Force Eddy Current Testing (LET) for non-contact measurements of the velocity of a conducting liquid and for detection of defects in the interior of solid bodies. The present, simplified configuration provides important scaling laws and reference results for *complete* numerical simulations. The results of computations are also compared with analytical solutions for an infinite plate.

Introduction

Lorentz Force Velocimetry (LVF) is a modern, contactless technique for measuring flow rates and velocities of moving conducting liquids. It can be used in situations where mechanical contact of a sensor with the flowing medium must be avoided due to high temperatures and chemical reactions [1]. Possible applications include flow measurement in a Submerged Entry Nozzle during the continuous casting of steel, in ducts and open channel flows of liquid aluminium alloys in aluminium production [2], and in other metallurgical processes where hot liquid metal or glass flows are involved. Eddy Current Testing (ECT) can serve as a basic tool for detecting sub-surface defects (cracks) in metallic constructions where these defects are critical for safety, e.g. air and railroad transport, engines, bridges, etc.

At the origin of LFV and ECT is Lenz' rule of magnetic induction. Its use for flow rate measurement was already proposed in [3]. Eddy currents are induced in a conductor which is moving in a (primary) magnetic field. The interaction of these eddy currents with the primary magnetic (Fig. 1) field creates a force that opposes the motion by

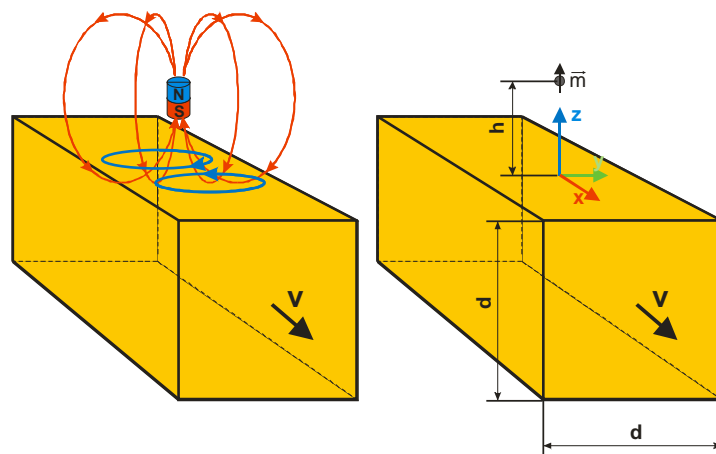


Fig. 1. Sketch and geometry parameters of the studied problem

Lenz' rule. The magnetic system, which creates the primary magnetic field, experiences a drag force acting along the direction of the conductor motion. Simple estimations show that this force is $F \sim \sigma v B^2$, where σ is the electrical conductivity of the moving conductor, $v=v_x=const$ is the velocity and B is the magnetic induction. Measuring this force acting on the magnetic system, allows us to measure the velocity of the moving conductor with high precision.

The drag force increases quadratically with the magnetic induction. This allows one to increase the sensitivity of this measurement technique by increasing the magnetic field intensity, whereby it can, in principle, be applied to poorly conducting bodies like electrolytes, ceramics or glass melts. However, this still requires research on the proper magnetic system design and optimization as well as an accurate and advanced force measurement system.

In general, one cannot find an analytical solution for the force acting on a realistic magnet system even when the motion of the conducting body is very simple. Only a few cases, which replace the real magnetic system by the magnetic dipole or simple coil, are known to have analytical solutions [4, 5]. However, these simplified problems are very important for LFV theory, because they allow a better understanding of the physics and provide reference data for complex numerical simulations. In the present paper, we consider a moving rectangular conducting bar in a field of the magnetic dipole, which represents a canonical problem for LFV theory. It generalizes the case of an infinite conducting plate, which can be treated analytically [5]. Its solution can be directly compared with results obtained from LVF and ECT applications for duct flows and solids without defects.

1. Mathematical Formulation of the Problem and Numerical Procedure

We consider an electrically conducting non-magnetic infinite solid bar with square cross-section $d \times d$ (Fig. 1), which is moving with a constant velocity v in x direction in the field \vec{B} of a magnetic dipole with the magnetic dipole moment \vec{m} . If the origin of the coordinate system corresponds to the dipole location then the field of the dipole at distance \vec{r} can be expressed as [6]:

$$\vec{B} = \frac{\mu_0}{4\pi} \left(\frac{3(\vec{m} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3} \right). \quad (1.1)$$

Eddy currents are induced in the bar when it crosses the magnetic field lines. They create a secondary magnetic field \vec{b} . In this work, we assume that the magnetic diffusion time is small and we can neglect the secondary magnetic field with respect to the primary magnetic field, i.e. the quasi-static approximation [7] is applied. It also means that for selected ranges of velocity v , electric conductivity σ and length scale h the magnetic Reynolds number $R_m = \mu_0 \sigma v h$ is small. The motion of the bar is prescribed, i.e. we consider the kinematic problem.

For the analysis we will use non-dimensional units based on the characteristic length $L_0 = d$, characteristic velocity equal to the bar velocity $V_0 = v$ and the characteristic magnetic field intensity $B_0 = \frac{\mu_0 m}{L_0^3}$. This selection of characteristic parameters leads to the following

expressions for the current density $\vec{j} = \sigma B_0 \vec{j}^*$ and Lorentz force $\vec{F} = \sigma v B_0^2 L_0^3 \vec{F}^*$ where 'star' symbol represents non-dimensional quantities (omitted below).

In the quasistatic approximation, the electric field can be represented as the gradient of the electrical potential ϕ . The induced current density can be expressed by Ohm's law for moving conductor as:

$$\vec{j} = -\nabla\phi + \vec{v} \times \vec{B}. \quad (1.2)$$

The moving bar is electrically neutral and according to the conservation of electric charge, the induced currents should be divergence-free $\nabla \cdot \vec{j} = 0$. Since the magnetic field of the dipole is solenoidal [6] and the velocity distribution uniform, the electrical potential satisfies the Laplace equation:

$$\nabla^2 \phi = 0. \quad (1.3)$$

A solution of this equation is required to obtain eddy currents using equation (1.2). Appropriate boundary conditions (BC) require zero normal currents on all side surfaces of the bar $\vec{j} \cdot \vec{n} = 0$. We also require that the electrical potential and currents should vanish at the remote ends.

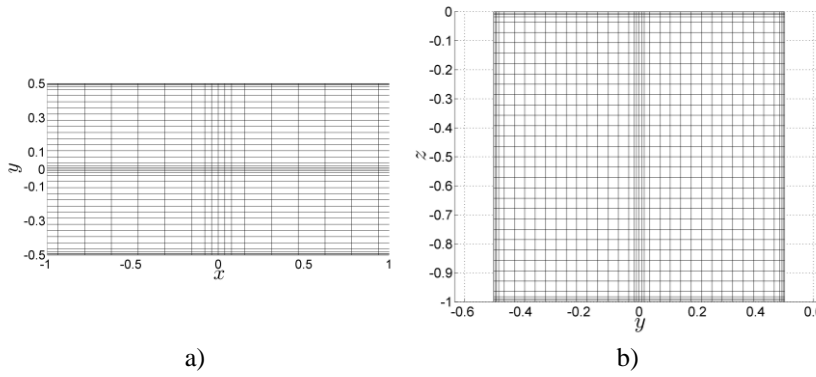


Fig. 2. Refined grid used for numerical simulations (the bar size is $7.5 \times 1 \times 1$): a) a central part for $z=const$, b) $x=const$

A general analytical solution of the problem with the described BC could not be obtained. For this reason, an automated *Matlab*TM script coupled with the *Comsol*TM FEM Laplace 'pardiso' solver [8] was used to solve it numerically for the electrical potential using second order Lagrangian elements [9]. The Lorentz force

was later computed taking the volume integral of its density $\vec{F} = \int_V \vec{j} \times \vec{B} dV$ or using the Biot-

Savart law for secondary field computation at location of the dipole. The force and torque then can be found as $\vec{F} = (\vec{m} \cdot \nabla) \vec{b}$ and $\vec{T} = \vec{m} \times \vec{b}$ correspondingly. The integration procedure was implemented using built-in *Comsol*TM functions.

Our preliminary results showed that the accurate solution of the problem requires very fine grid in a zone of large magnetic field gradients. Therefore, a refined grid (Fig. 2) was used for simulations. We also assume that computational errors are acceptable if the distance between the dipole and the top surface of the moving bar h equals the doubled characteristic size of the element. The computational grid was further refined for very small $h \approx 8 \cdot 10^{-2}$. The maximal number of elements in the grid was around 10^5 .

2. Results and Discussion

The numerical results for the moving bar are compared to the analytically obtained Lorentz force $F_x^0(h)$ for the translating infinite plate of identical thickness, where the dipole is vertically oriented [5]:

$$F_x^0 = \frac{1}{128\pi h^3} \cdot \left(1 - \frac{h^3}{(1+h)^3}\right). \quad (2.1)$$

Tab. 1. Selected values of the F_x/F_x^0 curve for [0,0,1] oriented dipole

h	0.08	0.20	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20	4.00
F_x/F_x^0	1.00	0.98	0.91	0.65	0.42	0.28	0.19	0.13	0.10	0.07	0.04

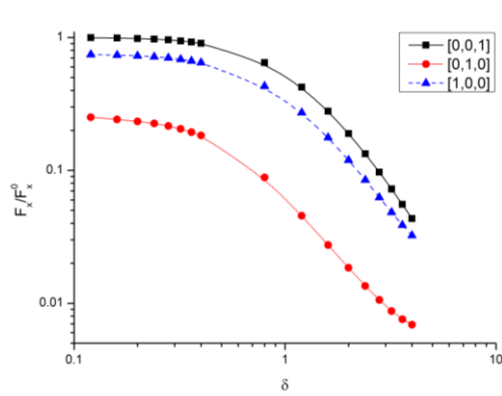


Fig. 3. The ratio of rectangular cross-section bar and infinite plate x force components acting on the magnetic dipole which is placed in the middle above the top surface for different dipole orientations

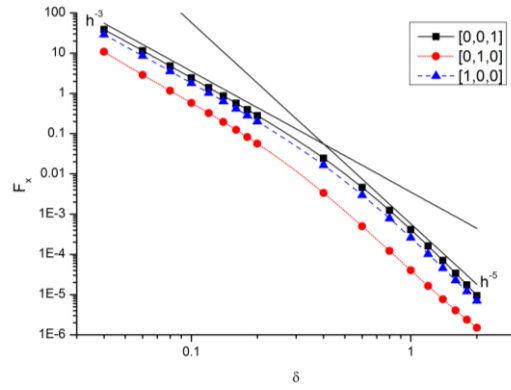


Fig. 4. Non-dimensional x force component acting on the magnetic dipole which is placed in the middle above the top surface for different dipole orientations

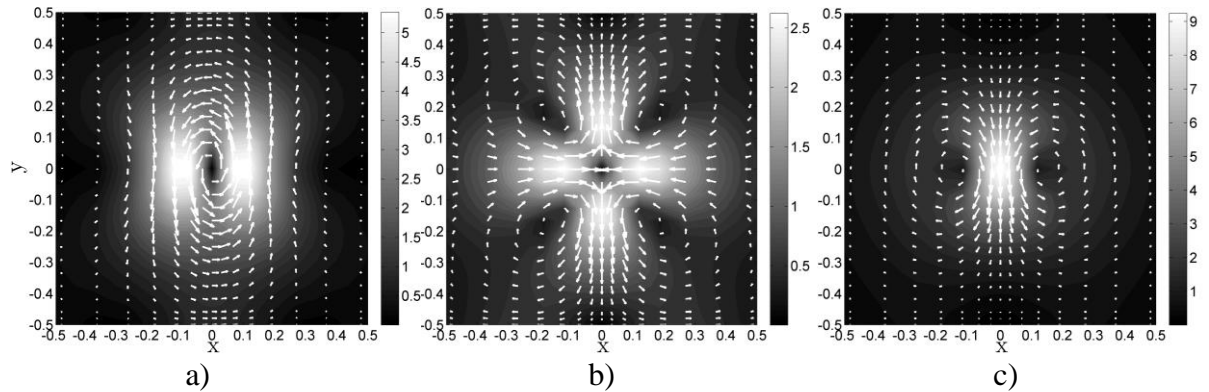


Fig. 5. Contours of non-dimensional current density magnitudes and current density vectors on the top surface of the bar for different orientations of the magnetic dipole: a) [1,0,0], b) [0,1,0], c) [0,0,1]. Only central part of the top surface is shown

For this case, the magnetic moment components are [0,0,1] in our non-dimensional representation. We consider three different orientations [1,0,0], [0,1,0] and [0,0,1]. Fig. 3 and

Tab. 1 present the ratio F_x/F_x^0 and Fig. 4 the non-dimensional force F_x . The force ratio curve F_x/F_x^0 is decreasing monotonously when h is increased as expected. When h tends to zero, i.e. the magnetic dipole approaches the top surface of the bar, the curve reaches unity for the vertically oriented magnetic dipole. This result conforms to expectations because the rectangular bar acts on the dipole exactly as the infinite plate if the dipole approaches its surface. If the dipole is moved to a new position away from the bar then the force ratio starts to decay until it reaches very small values

because the “useful” volume under the influence of the magnetic dipole field decreases for the rectangular bar when compared with the infinite plate. The part of the curve for $h > 2$ can be approximated by a power law. A fit to the data provides $F_x/F_x^0 = 0.76 h^{-2}$. The same power law approximation for F_x (Fig. 4) gives $F_x \sim h^{-3}$ for small h and $F_x \sim h^{-5}$ for large h . Equation (2.1) provides the estimate $F_x^0 \sim h^{-4}$ for the infinite plate, i.e. the Lorentz force decays faster for moving solid bar than for infinite plate.

The other orientations of the magnetic dipole aligned with the coordinate system axes provide smaller values of the Lorentz forces comparing to the infinite plate. The Lorentz force on a dipole oriented in y direction is approximately $1/4$ part of the force for vertically oriented dipole because the magnetic field intensity is twice smaller and $F_x \sim B^2$. If the dipole is oriented in x direction, then the force can reach 75% of the value obtained for vertically oriented dipole. This can be explained by the current density distribution in the bar (Fig. 5). It can be seen that the current density magnitude for $[1,0,0]$ oriented dipole is higher than for $[0,1,0]$ oriented dipole and the current density maximum is better localized in the region of the highest magnetic field intensity for $[1,0,0]$ oriented dipole.

It is often required to increase the sensitivity of the LFV and the simplest way to make it is to symmetrically introduce a second magnet from the other side (assuming that we cannot decrease h or increase m) which in our case is represented as the second magnetic dipole with the same magnetic moment magnitude and the distance from the bar's closest surface. Both dipoles have no shift in y direction and their magnetic moments are collinear to z axis.

The numerical solution shows (Fig. 6) that the only one way to increase the Lorentz force is to place the both dipoles unidirectionally. It can be seen that for small h values the magnetic field of each dipole is localized at the closest surface, i.e. both magnetic dipoles work independently and we simply find a doubling of the force. With increasing h , the magnetic field is less localized and the Lorentz force increases if the dipoles have unidirectional orientation or decays if the orientation is opposite. It is expected that the addition of the second magnetic dipole will introduce four times greater Lorentz force for large h . This tendency can be clearly seen in Fig. 6.

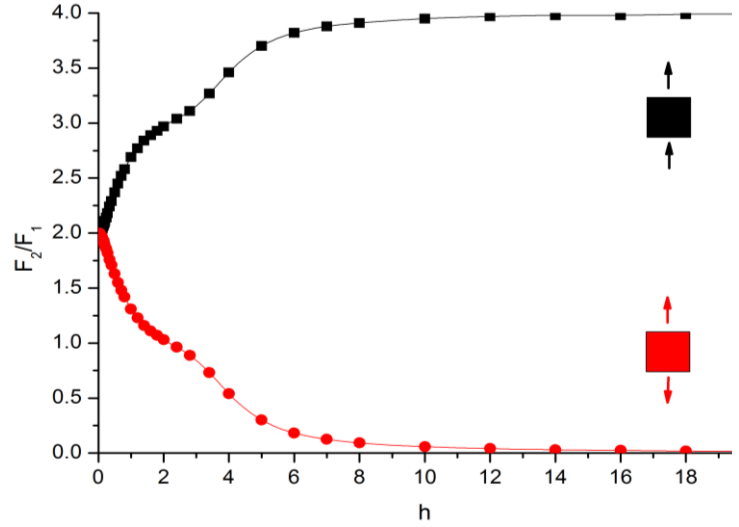


Fig. 6. The ratio between the force acting on two dipoles and the force acting on one magnetic dipole

Conclusions

The kinematic problem of a translating solid conducting body under the influence of a magnetic dipole has been investigated, where the dipole is located in the lateral mid-plane of the bar. The results show that the maximal Lorentz force can be obtained for the magnetic dipole oriented in the vertical direction. The force depends on the distance h between dipole and bar. It is given by power laws when the distance h is small or large compared with the width of the bar. For small distances, the power law is identical to the case of an infinite plate. At large distance, the power law for the bar shows a more rapid decay than for the infinite plate. The proposed computational method can be applied to cylindrical solid body translation and to investigation of the kinematic problem of a laminar conducting flow.

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