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Dipole Aproximation Limits for Magnetic Interaction Forces Between Spheres

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Abstract

The interaction between ferromagnetic particles plays the main role in behaviour of a magnetorhelogical suspension (MRS). The present paper describes dipole approximation limits for magnetic forces between spherical particles in 2D axisymmetric case. Nonlinear magnetization of particles is described by Froelich-Kennely law.

Introduction

Magnetorheological suspensions are used in many devices where controllable friction is needed. Optimisation of devices with MRS requires correct description of the magnetic interaction force between ferromagnetic particles, usually spherical ones. A natural choice would be to use dipolar interaction model, regretfully close particles behave differently from point-dipoles. In this case finite element approach provides numerical values of a magnetic force, taking into account nonlinear magnetization of the ferromagnetic material. Here attempt is made to estimate limits of validity of point-like dipole approximation for magnetisable spheres. The dipolar approximation is realized via “Lonely sphere” model which corresponds to single sphere in an external homogeneous magnetic field with constant magnetization. Such a sphere mathematically is equivalent to point like dipole placed in the centre of sphere with the same dipolar moment as the original sphere.

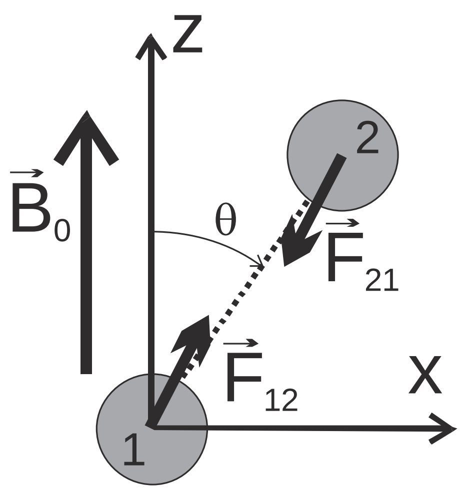


Fig. 1. Magnetic interaction of two particles

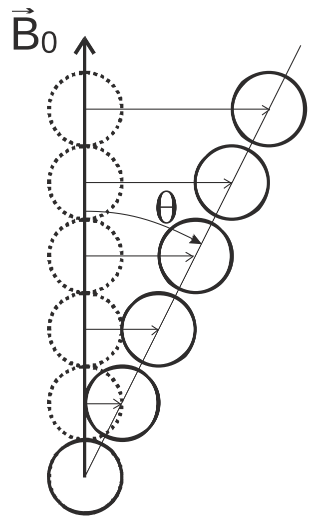


Fig. 2. Affine shear of chain

One of the main outcomes of simple models is the estimation of yield stress value [1,2], providing important macroscopic property of MRS.

1. The Magnetic Force Between Spheres

The magnetic force between dipoles (Fig.1.) is different from the electric forces between electric charges because it is not “central force”. The value and direction of force is given by formula:

$$\vec{F}_{12} = \frac{3\mu_0}{4\pi R_{12}^5}\left((\vec{m}_1 \cdot \vec{R}_{12})\vec{m}_2 + (\vec{m}_2 \cdot \vec{R}_{12})\vec{m}_1 + (\vec{m}_1 \cdot \vec{m}_2)\vec{R}_{12} - 5(\vec{m}_1 \cdot \vec{R}_{12})(\vec{m}_2 \cdot \vec{R}_{12})\frac{\vec{R}_{12}}{R_{12}^2} \right). \tag{1.1}$$

Here $\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$ is distance vector between the second and the first sphere (Fig.1.), $\vec{m}_1$, $\vec{m}_2$ are dipole vectors of spheres, μ_0 is magnetic constant.

So called “Lonely sphere” model describes single sphere placed in an external magnetic field $\vec{B}_0$. Magnetization of a single sphere is

$$\vec{M} = \frac{3(\mu - 1)}{(\mu + 2)}\frac{B_0}{\mu_0} = \frac{3(\mu - 1)}{(\mu + 2)}H_0. \tag{1.2}$$

Since magnetic permeability $\mu = \mu(H)$ is function of magnetic field, use of (1.2) is not easy. Solving relation

$$\vec{H}_{in} = \frac{\vec{B}_{in}}{\mu\,\mu_0} = \frac{3}{(\mu + 2)}\frac{B_0}{\mu_0}, \tag{1.3}$$

as equation for magnetic permeability together with Froelich-Kennely law [3]

$$\mu(H) = 1 + \frac{(\mu_{ini} - 1)M_S}{M_S + (\mu_{ini} - 1)H}, \tag{1.4}$$

provides magnetic field intensity inside the sphere:

$$H_{in} = \frac{3H_0(\mu_{ini} - 1) - M_S(\mu_{ini} + 1) + \sqrt{36H_0 M_S(\mu_{ini} - 1) + (3H_0(\mu_{ini} - 1) - M_S(\mu_{ini} + 1))^2}}{6(\mu_{ini} - 1)}. \tag{1.5}$$

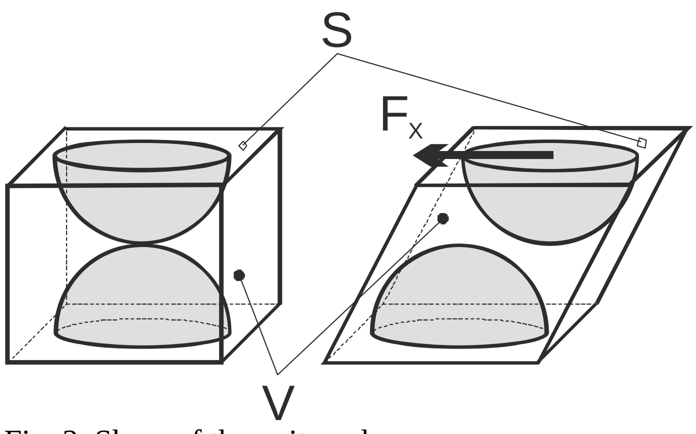
Now we can calculate magnetic moment of the magnetized sphere with radius R_0 placed in an external field $\vec{B}_0$ and using formulae (1.2), (1.5)

$$\vec{m} = \frac{4\pi R_0^3}{3}\vec{M} = \frac{4\pi R_0^3}{3}(\mu(H_{in}) - 1)H_{in}. \tag{1.6}$$

(1.5)

2. Estimation of the Yield Stress

The value of the yield stress is calculated by simple model of the chain exposed to affine shear (see Fig.2). The aim is to find the maximal value of the “tangential force” which acts 6n the half of the chain. The value of the force is taken from FEM calculations at given



$\vec{B}_0$ and separation $d = 2R_0\left(\frac{1}{\cos\theta} - 1\right)$ between spheres. The maximal value of tangential projection of force $F_X = \sin\theta \, F(B_0, d)$.

The value of the yield stress for given volume fraction ϕ of particles is calculated by

Fig. 3. Shear of the unite volume

$$\tau = \frac{F_X}{S} = \frac{3\phi}{2\pi}\frac{F_X}{R_0^2}. \qquad (1.7)$$

Volume fraction is related to unit cell (Fig.3.) volume V and upper surface area S as

$$\phi = \frac{V_{Sphere}}{V} = \frac{4\pi R_0^3/3}{2R_0 S} = \frac{2\pi R_0^2}{3S}. \qquad (1.8)$$

3. Finite Element Method

Finite element calculations are carried out using azimuthal component of vector potential, all details are given in [4]. Equation to solve is elliptic second order differential equation:

$$\frac{\partial}{\partial r}\left(\frac{1}{\mu}\frac{\partial A}{\partial r} + \frac{A}{\mu r}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\mu}\frac{\partial A}{\partial z}\right) = 0. \qquad (1.9)$$

Boundary conditions are $A|_{r=0} = 0$ on axis ($r = 0$), $\frac{\partial A}{\partial z} = 0$ on symmetry boundaries ($z = const$) and $A| = \frac{B_0 r}{2}$ on “far” boundary.

4. Results

Comparison of FEM calculated force values and dipolar approximation ones (using Lonely sphere model) are shown on Fig.4. FEM calculations were carried out using 1st and 2nd order element. Values $\frac{f\_1st}{f\_dip}$ (ration of FEM 1st order element force versus dipolar one) and $\frac{f\_2nd}{f\_dip}$ (ration of FEM 2nd order element force versus dipolar one) are plotted for there values $B_0 = 0.1;\ 1;\ 10$. It is evident that for $d/R_0 < 5$ FEM provide reliable results which are generally quite different from dipolar approximation. Increase of the distance between particles leads to less accurate FEM calculated values, for $d/R_0 > 10$ 1st order elements are not accurate enough. General recommendation is to use dipolar approximation for distances larger that 5 sphere radiuses.

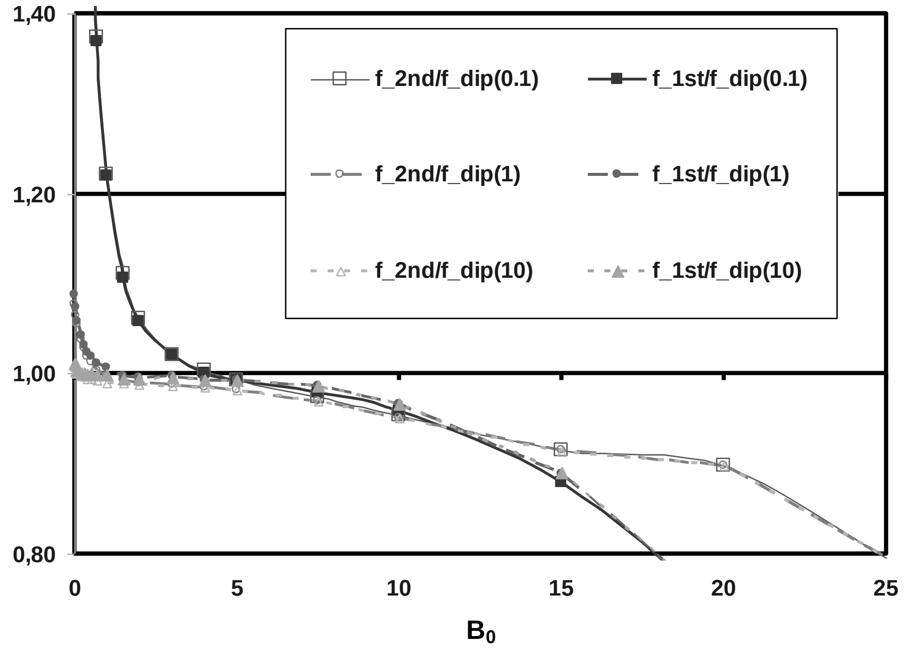


Fig. 4. Ratio of FEM calculated versus dipolar approximation force

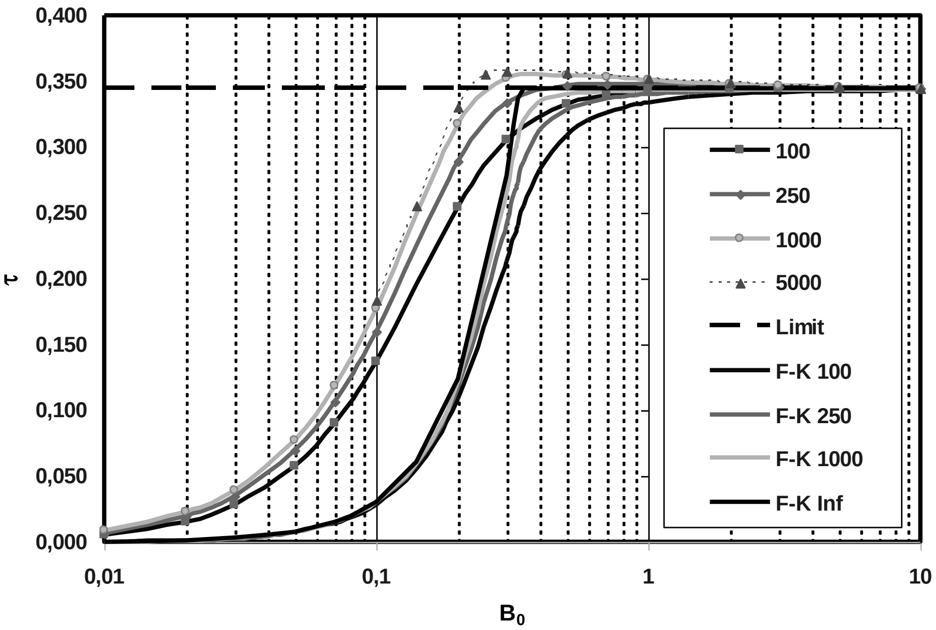


Fig. 5. Rupture force

In Fig.5. so called non-dimensional rupture force, related to yield stress is plotted versus B_0. Curves with markers correspond to FEM 2nd order calculations at different values of μ_{ini} (100, 250, 1000, 5000). Solid curves are obtained by dipolar approximation (Lonely sphere model). FEM and dipolar forces are provide completely different results. Just at saturation all curves tend to the theoretically predicted non-dimensional value 0.344. Thus it is obvious, that 2D axially symmetric FEM calculations provide yield stress results with completely different behaviour than simple dipole approximation.

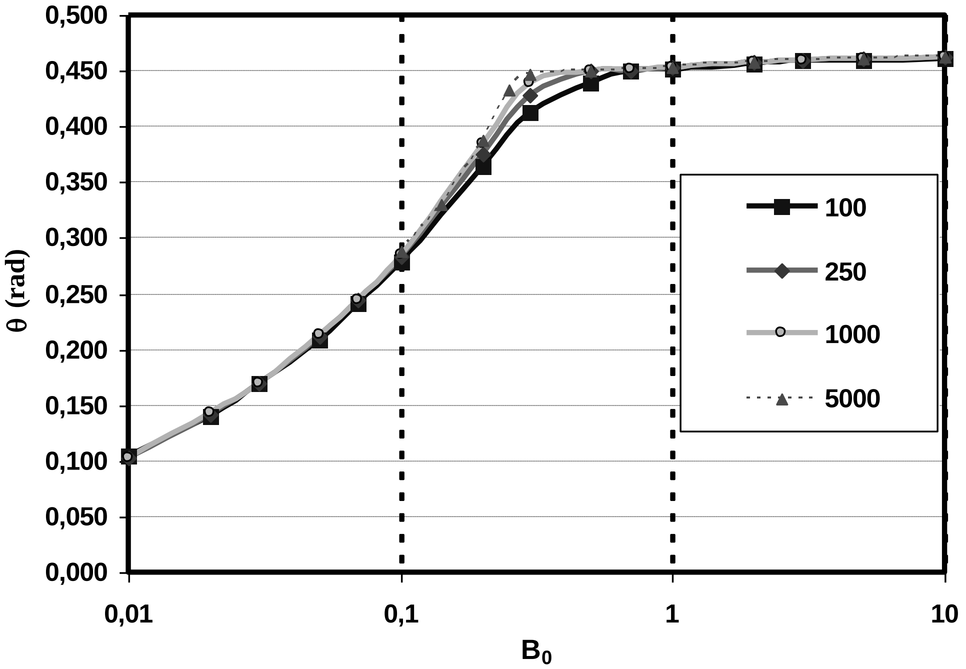


Fig. 6. Rupture angle

On Fig.6. rupture angle (angle θ in Fig.2.) is shown. It increases with increase of magnetic field till the magnetization of particles reach saturation, the limiting value $\theta \approx 0.452\,rad$.

On Fig. 7. rupture force is plotted as log-log plot indicating $\propto (B_0)^{1.5}$ type behaviour for FEM results at small magnetic field values. Results obtained at constant magnetic permeability μ is hard to compare with nonlinear magnetization, dependence on magnetic field value is like $\propto (B_0)^2$ as it should be for a linear task due to the magnetic force scaling.

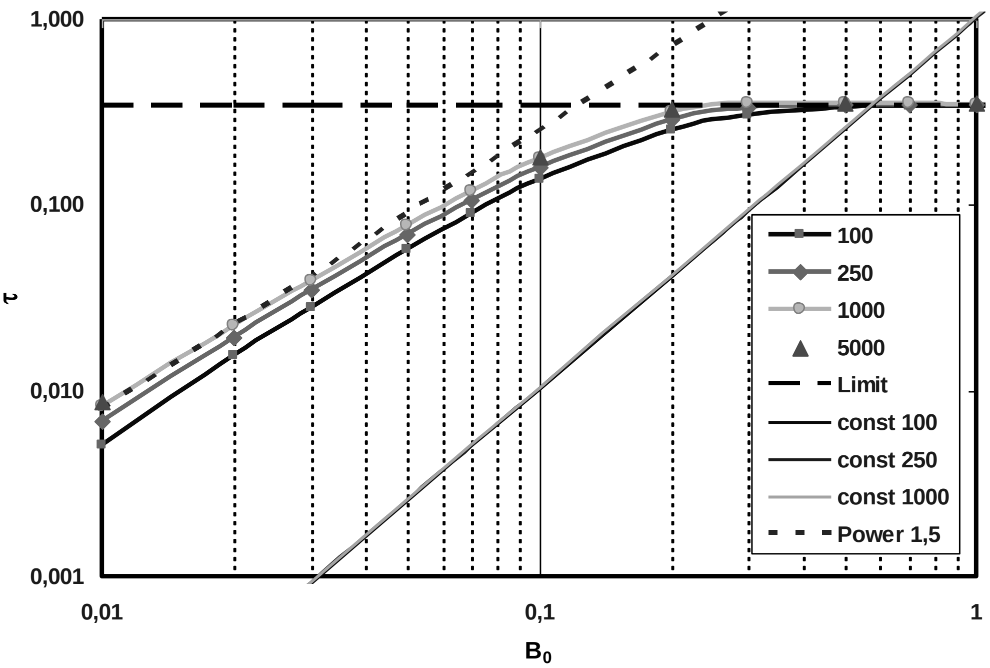


Fig. 7. Rupture force at constant μ

Conclusions

Results confirm that 2nd order finite elements provide more accurate results, especially for large distances between particles. FEM calculations are needed for distances between particles up to 5 sphere radiuses ($d < 5R_0$). For larger distances numerical errors accumulate and accuracy of FEM calculations reduces and dipolar approximation provides better result with less computational effort, assuming, that "Lonely sphere" model is used.

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