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Simplified Dynamic Mathematical Model for Heat Transfer by Radiation and Heat Conduction

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Abstract

The present material is focused on modelling of complex system simplified heat exchange dynamics. Stationary solutions of chosen problem are described. Possible methods for heat problem solution are lined out. Chosen approach of small deviation modelling is explained. General conclusions about application possibilities and restrictions are given.

Introduction

In modelling of real life heat exchange problems, it must be taken into account that modern devices are very complicated (usually consisting of many parts). Correct heat exchange analytic description of very complicated device is inconvenient, numerical calculations are used instead. Stationary solutions for very complicated devices are common in numerical calculations. Despite the huge progress of computational power, unsteady numerical simulations for such complicated devices are too demanding on computational capability and therefore practically are not usable.

Nevertheless there are situations when dynamic calculations of heat exchange process are necessary. Therefore simplified description of heat exchange should be introduced. From literature follows that simplified description of heat transfer in several systems has already been developed, see list of references [1]. Nevertheless such models can be used only for very simple system geometry.

It was concluded, that in order to create simplified description of dynamic behaviour of complicated heat exchange system, stationary solution close to interesting heat parameter values can be used as a base (because correct stationary solution is not a problem in terms of computational power). In order to reduce error due to simplifications of physical model, only small deviations from stationary solution are described.

1. Concept of Using Stationary Solutions and Small Deviations

Because in reality used technological devices for heating, melting, welding etc are very complicated, the corresponding numerical simulations are quite complex. To ease the task for special applications only stationary simulations are carried out. In stationary solution it is assumed that temperature field all over the selected device is constant and heat flows remains also unchanged.

After stationary calculation has been done (with any software, that is able to compute stationary result of heat exchange problem), various chosen device characteristic parameters (temperature, heat flows etc., arbitrary system parameter will be denoted as F) have been obtained. It can be safely assumed that computed stationary parameter value F_{stat} is not changing over time. Schematic representation is given in Fig. 1.

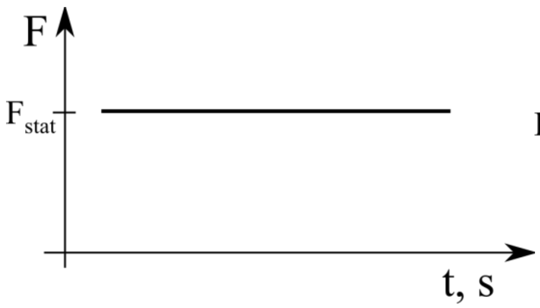

Fig. 1. Schematic representation of small deviations from parameter stationary value

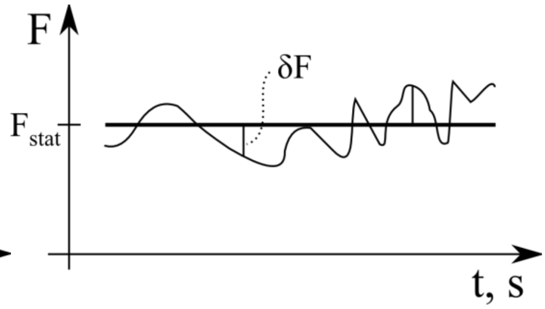

Fig. 2. Schematic representation of arbitrary parameter stationary value

In reality stationary case of process is quite rare. Usually there would be deviations from stationary parameters even if one would try to keep system state close to stationary solution. The cause of this deviation can be very different, from imperfect heat power system, up to fluctuations introduced from system mechanical dynamics. Therefore, to model real situation, one must describe deviations from stationary solution parameters. Schematic representation of chosen parameter is shown in Fig. 2.

In order to describe such small deviations from stationary value (Fig. 2), one must create a physical description. Example with arbitrary function and two arguments is shown in equation (1). For derivation of parameter deviation full differential expression (2) can be written. If any unknown parameter is left, stationary solution correspondence to physical description can be used, equation (3).

$$F = f(x_1, x_2); \quad (1)$$

$$\delta F = \frac{\partial f}{\partial x_1}\delta x_1 + \frac{\partial f}{\partial x_2}\delta x_2; \quad (2)$$

$$F_{stat} = f(x_{1,stat}, x_{2,stat}) . \quad (3)$$

2. Examples of Small Deviation from Stationary Solution Usage in Heat Transfer

2.1. Heat Conduction

In order to keep heat transfer description simple, we will consider the system for which two chosen objects are assumed to each have approximately constant temperature over all volume. In heat conduction case, thermal insulation is placed between two chosen objects. It also must be noted, that objects and insulation can be of any arbitrary shape. Schematic representation of described situation is given in Fig. 3.

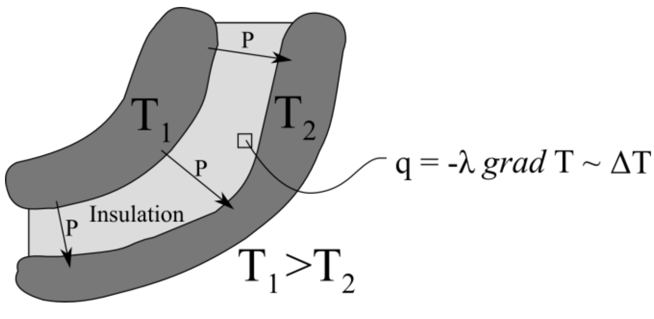

Fig. 2. Schematic representation of heat conduction for two arbitrary shaped constant temperature bodies

It must be reminded, that for each small volume part of insulation, conducted heat flow density is described with Fourier law (physical laws for heat transfer can be found in list of references [2]). This local description is dependent both from shape of insulation and heat conductivity in insulation.

It can be concluded that total transferred heat power is dependent from two object temperature difference, simplified expression can be written as follows (4):

$$P = k\left(T_1 - T_2\right), \tag{4}$$

where coefficient k describe given situation, dependent from different heat conductivity λ values over insulation volume and also dependent from specific geometry of objects and insulation. Coefficient k value is not precisely known from physical description yet.

As it was explained in section before, stationary parameter values also must follow the same general physical description, in this case expression (4). Therefore if one can obtain stationary parameter P_{stat}, $T_{1,stat}$ and $T_{2,stat}$ values (from numerical stationary calculations; it must be noted, that P_{stat} value is determined from integration over surface, temperature $T_{1,stat}$ and $T_{2,stat}$ value determination should be as average over volume; if such numerical determination is not possible, temperature values could be obtained with different creative approach), coefficient k can be derived as shown below (5 – 6):

$$P_{stat} = k\left(T_{1,stat} - T_{2,stat}\right); \tag{5}$$

$$k = \frac{P_{stat}}{T_{1,stat} - T_{2,stat}}. \tag{6}$$

If one has found the unknown coefficient k value (6), final expression for heat flow deviation δP trough insulation can be written (for case, when heat conduction is not dependent from temperature, Fig. 3), derivation and result is shown below (7):

$$\delta P_{cond} = \frac{\partial P}{\partial T_1}\delta T_1 + \frac{\partial P}{\partial T_2}\delta T_2 = k\delta T_1 - k\delta T_2 = \frac{P_{stat}}{T_{1,stat} - T_{2,stat}}\left(\delta T_1 - \delta T_2\right), \tag{7}$$

where δT_1 and δT_2 are respective deviations in object temperatures (objects can be seen in Fig. 3). Acquired expression (7) is a convenient way to calculate change in heat flow, if one knows changes in respective object temperatures.

2.2. Heat Radiation

It is assumed that the considered object has approximately constant temperature over its surface and that the object is placed far from other objects; therefore no radiation back from other objects is present. Again it also must be noted, that object can be of any arbitrary shape, see Fig. 4. It must be reminded, that for each small surface part of object, radiated heat flow density is described with Stefan-Boltzmann law. It can be concluded that total transferred heat is dependent from object characteristic temperature:

$$P = kT_1^4, \tag{8}$$

where coefficient *k* describe given situation, dependent from specific geometry and also small deviations in temperature field. Coefficient k value is not precisely known from physical description yet.

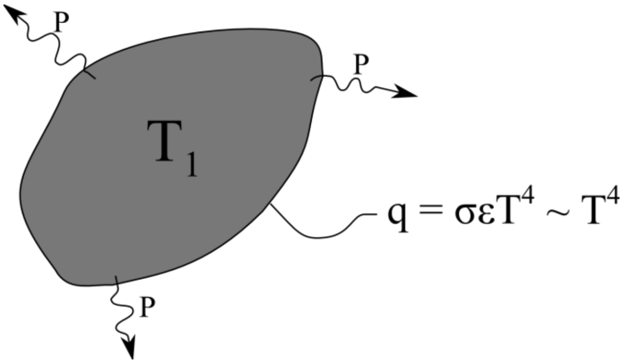

Fig. 3. Schematic representation of heat radiation for one arbitrary shaped constant temperature body

As it was explained in section before, stationary parameter values also must follow the same general physical description, in this case expression (8). Therefore if one can obtain stationary parameter P_{stat} and $T_{1,stat}$ values (from numerical stationary calculations), coefficient *k* can be derived as shown below (9 – 10):

$$P_{stat} = kT_{1,stat}^4; \tag{9}$$

$$k = \frac{P_{stat}}{T_{1,stat}^4}. \tag{10}$$

If one has found the unknown coefficient *k* value (10), expression for heat power deviation δP radiated away from object surface can be written, derivation and result is shown below (11):

$$\delta P_{rad} = \frac{\partial P}{\partial T_1}\delta T_1 = 4kT_1^3\delta T_1 = \frac{4P_{stat}}{T_{1,stat}}\delta T_1, \tag{11}$$

where δT_1 is respective deviation in object temperature (object can be seen in Fig. 4). Characteristic stationary temperature value in this case can be obtained form Stefan-Boltzmann law (in order to average temperature field), relation for stationary temperature value and final expression for heat power radiated away from object is shown below (12 – 13):

$$T_{1,stat} = \sqrt[4]{P_{stat}/(\sigma\varepsilon S)}, \tag{12}$$

$$\delta P_{rad} = \frac{4P_{stat}}{\sqrt[4]{P_{stat}/(\sigma\varepsilon S)}}\delta T_1, \tag{13}$$

where ε is emissivity of selected object surface (assumed to be constant) and *S* is object surface area. Acquired expression (13) is a convenient way to calculate change in radiated heat power, if one knows changes in respective object temperature.

3. Application of Developed Heat Radiation and Conduction Description for Calculation of Transient Heat Exchange Process in Selected System

In order to better illustrate described small deviation approach based on stationary solution to model heat exchange dynamics, a simple heat exchange system is introduced. The system in question consists of heater, insulation around heater and object to be heated by heat radiation from the heater. Schematic representation is given in Fig. 5.

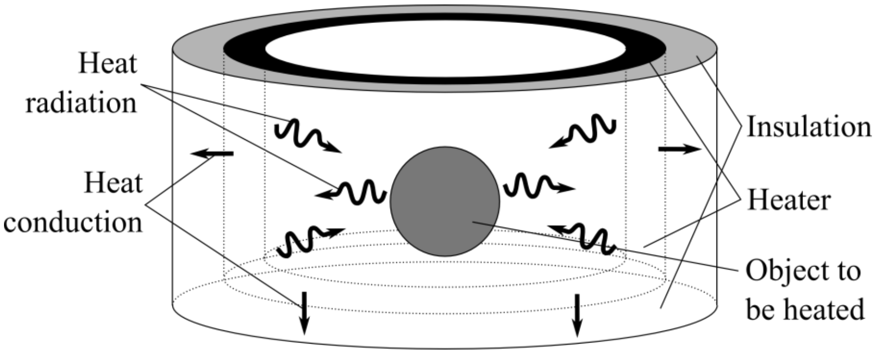

Fig. 4. Schematic representation of chosen heat exchange system

As it was described before, modelling of heat exchange dynamics is based on stationary solution. Therefore in beginning, before dynamic calculations of heat exchange process, stationary solution close to our interesting parameter (for dynamical calculations) range must be found. From stationary calculations heat powers $P_{i,cond,stat}$, $P_{i,rad,stat}$ and temperature values $T_{i,stat}$ can be found for all objects and surfaces.

In order to further describe system in question, objects of main interest must be lined out. In this case one can assume that main objects are heater and object to be heated (Fig. 5). For those objects also total heat capacity $c_p m$ [J/K] (specific heat capacity at constant pressure selected due to reason, that it is the most probable case) must be determined. As next step heat balance equations for objects in interest must be created (14 – 15):

$$\text{Heater: } c_{p,h} m_h \frac{\partial T_h}{\partial t} = P_h + P_{cond,h} + P_{rad,h}, \quad (14)$$

$$\text{Object to be heated: } c_{p,h} m_o \frac{\partial T_o}{\partial t} = P_{cond,o} + P_{rad,o}, \quad (15)$$

where P_h is heater power (usually electric), whereas P_{cond} and P_{rad} are heat powers transferred trough conduction and radiation processes. It must be noted, that object has no direct power source, and therefore its temperature change is defined solely by heat transfer. Expression shown above (14 – 15) represents full heat balance equations for general case. For small deviations around stationary solution equations are similar (16 – 17):

$$\text{Heater: } c_{p,h} m_h \frac{\partial \delta T_h}{\partial t} = \delta P_h + \delta P_{cond,h} + \delta P_{rad,h}, \quad (16)$$

$$\text{Object to be heated: } c_{p,h} m_o \frac{\partial \delta T_o}{\partial t} = \delta P_{cond,o} + \delta P_{rad,o}, \quad (17)$$

where δF is respective arbitrary parameter F small deviation around stationary solution. Bearing in mind the simple examples of heat conduction and heat radiation small deviation description shown before, schematic final expressions for change of temperature deviations can be written (18 – 19):

$$c_{p,h}m_h\frac{\partial \delta T_h}{\partial t}=\delta P_h-\left(\sum_i\frac{P_{stat,cond,i}}{\Delta T_{stat,i}}+\sum_j\frac{4P_{stat,rad,j}}{T_{stat,j}}\right)\delta T_h+\left(\sum_i\frac{P_{stat,cond,i}}{\Delta T_{stat,i}}+\sum_j\frac{4P_{stat,rad,j}}{T_{stat,j}}\right)\delta T_o \quad (18)$$

$$c_{p,h}m_o\frac{\partial \delta T_o}{\partial t}=-\left(\sum_i\frac{P_{stat,cond,i}}{\Delta T_{stat,i}}+\sum_j\frac{4P_{stat,rad,j}}{T_{stat,j}}\right)\delta T_o+\left(\sum_i\frac{P_{stat,cond,i}}{\Delta T_{stat,i}}+\sum_j\frac{4P_{stat,rad,j}}{T_{stat,j}}\right)\delta T_h. \quad (19)$$

It must be noted that expressions (18 – 19) are regrouped in respect to expressions (16 – 17) in order to better represent the meaning of different parts. The coefficient before object temperature itself represents heat losses due to increase of object temperature, whereas coefficient before other objects (this case only one) represents heat gains due to increase of other objects.

Conclusions

The simplified non-stationary heat exchange description based on small temperature deviations can be used for the description of heating systems which have oscillations around the steady state. The steady state itself can change in time also with characteristic transition time much larger than characteristic time of considered oscillations. It has been numerically verified, that described simplified approach works with acceptable precision, if temperature field in constant temperature assumption for each body area does not change more than 10%.

Acknowledgement

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References

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