International Scientific Colloquium
Modelling for Material Processing
Riga, June 8-9, 2006

Physics of Elastic Magnetic Filaments

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Abstract

The model of an elastic magnetic filament is developed. Model of elastic magnetic filament allows investigating dependence of dynamics of filaments on several physical parameters: magnetoelastic number *Cm*, frequency of magnetic field, coefficient of friction etc.

By numerical simulation of the dynamics of filament shapes under action of magnetic field it is shown that the characteristic 'U' – like stable shapes (hairpins) can be formed. 'U' – like shapes of filament can exist in case of rotating magnetic field with small frequencies. If frequency increases 'U' – like shape relax to 'S' – like shape. Equations for describing critical frequency dependence on magnetoelastic number are obtained.

It is shown that in unsteady magnetic field flexible magnetic filament swims in direction of magnetic field.

Introduction

There is a great interest to develop chains of magnetic particles connected with polymer in order to use them in medicine [1, 2]. Elastic magnetic filaments can be found in nature [3] or created in laboratories [4, 5]. Such filaments can be used as micromechanical sensors [5, 6].

There is developed mathematical model [7, 8, 9] for understanding physical phenomena observed in experiments. First attempt to create theoretical model based on the extension of the Kirchhoff model of the elastic rod by including the magnetic energy term was done in [10].

In this paper a model of elastic anisotropic filament is developed. This model allows us to investigate influence of different physical parameters on dynamics of flexible elastic filament. The model predicts a critical frequency of a rotating magnetic field when the elastic rod turns no more synchronously with the field. It is investigate that there exist stable 'U' – like configurations besides 'S' – like configurations of filaments in rotating magnetic field. 'U' – like configurations are observed in experiments. As shown in present work it is not necessary to assume inhomogeneities of filament magnetization [11] to explain existing of 'U' – like configuration of filament in rotating magnetic field.

1. Model

As model for describing elastic chains from magnetic particles connected with polymer the Kirchhoff model of a non-stretchable elastic rod is chosen. The magnetic energy of the filament is accounted by slender-body approach locally approximating the magnetic filament as a long cylinder. For the dynamics of the magnetic filament in viscose medium there is chosen the simplest approach – Rose dynamics.

According to the Kirchhoff model of the elastic filament extended by including the magnetic term the energy of filament reads as

$$E = \frac{1}{2} C \int \frac{1}{R^2} dl - \frac{2\pi^2 a^2 \chi^2 H_0^2}{\mu + 1} \int \left(\vec{h} \cdot \vec{t}\right)^2 dl . \tag{1.1}$$

Here R is the radius of curvature of the centerline. C is expressed by the radius of filament a and Young modulus Y of filament as follows: $C = \frac{\pi}{4} a^4 Y$. $\mu = 1 + 4\pi\chi$, where χ is the magnetic susceptibility of the filament. $H_0 \vec{h}$ is the external magnetic field strength in the direction of the unit vector $\vec{h}$. $\vec{n}$ is the normal to the centerline of the filament. The constant magnetic field along x-axis is considered in model. The local nonstretchability of the filament is accounted by introducing the local tension of the filament as a Lagrange multiplier. The total energy in this case has the form

$$E = \frac{1}{2} C \int \frac{1}{R^2} dl - \frac{2\pi^2 a^2 \chi^2 H_0^2}{\mu + 1} \int \left(\vec{h} \cdot \vec{t}\right)^2 dl - \int \Lambda dl . \tag{1.2}$$

Considering the variation of (2) with respect to $\vec{r}' = \vec{r} + \vec{\xi}$, the following expression for the first variation of the energy functional is obtained:

$$\delta E = [M \delta\varphi] + [F_t \xi_t] + [F_n \xi_n] - \int K_n \xi_n dl - \int K_t \xi_t dl . \tag{1.3}$$

Here [ ] denotes the values at the ends of the filament, ξ_n and ξ_t are components of the Lagrange displacement in normal and tangent directions to the centerline of the filament respectively. $\delta\varphi = \frac{\partial \xi_n}{\partial l} - \frac{\xi_t}{R}$ is the angle of tangent rotation at the Lagrange displacement positions $\vec{\xi}$. The tangent and normal vectors are connected according to the Frent equation $\frac{d\vec{t}}{dl} = -\frac{1}{R}\vec{n}$, where l is arc length of the filament's centerline. The binormal $\vec{b}$ to the centerline is defined by $\vec{b} = [\vec{t} \times \vec{n}]$. According to the relation (3), the following expressions for the components of the body force $\vec{K}$, stresses $\vec{F}$ and momentum stresses $\vec{M} = M\vec{b}$ are valid:

$$F_n = C\left(\frac{1}{R}\right)_l + \frac{2\pi^2 a^2 \chi^2 H_0^2}{\mu + 1} Sin(2\vartheta), \tag{1.4}$$

$$F_t = -\frac{1}{2} C \frac{1}{R^2} - \frac{2\pi^2 a^2 \chi^2 H_0^2}{\mu + 1} Cos^2 \vartheta - \Lambda , \tag{1.5}$$

$$K_n = C\left(\frac{1}{R}\right)_{ll} + \frac{1}{2} C \frac{1}{R^3} + \Lambda \frac{1}{R} +$$

$$+ \frac{2\pi^2 a^2 \chi^2 H_0^2}{\mu + 1}\left(\left(Sin(2\vartheta)\right)_l + \frac{1}{R} Cos^2 \vartheta \right), \tag{1.6}$$

$$K_t = -\Lambda_l , \tag{1.7}$$

$$M = -C\frac{1}{R}. \tag{1.8}$$

In [10] solution of variation of energy functional (2) by author is done not correctly and one of terms are neglected, what leads to disagreement of equations (4 - 8) and (3 - 7) in [10]. It can be easily obtained that

$$\vec{K} = \frac{d\vec{F}}{dl} = \frac{d}{dl}\left(F_n\vec{n} + F_t\vec{t}\right). \tag{1.9}$$

In the simplest case of Rouse dynamics, when the hydrodynamic interaction between the particles in the chain is neglected, we have

$$\zeta\vec{v} = \vec{K}. \tag{1.10}$$

Here ζ is the friction coefficient of the filament per unit length. To consider anisotropy of the friction coefficient dimensionless coefficient β is introduced. This coefficient represents ratio between the friction coefficients in normal and tangential directions of the filament. In this case equation (10) reads as

$$\beta\zeta v_n = K_n, \tag{1.11}$$

$$\zeta v_t = K_t. \tag{1.12}$$

According to [12] a coefficient β can be set from *1,5* to *2*. According to the inextensibility of the filament

$$\frac{\partial v_t}{\partial l} + \frac{1}{R}v_n = 0, \tag{1.13}$$

and the relation for tangent angle change

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial v_n}{\partial l} - v_t\frac{1}{R}. \tag{1.14}$$

Following set of dimensionless equations for the tension Λ in the filament and tangent angle ϑ is obtained

$$\vartheta_l^2\Lambda - \beta\Lambda = -\vartheta_l\left(\vartheta_{lll} + \frac{1}{2}\vartheta_l^3\right) + Cm\vartheta_l\left(\left(Sin(2\vartheta)\right)_l - \vartheta_l Cos^2\vartheta\right), \tag{1.15}$$

$$\beta\vartheta_t = -\vartheta_{llll} - \frac{1}{2}\left(\vartheta_l^3\right)_l + Cm\left(Sin(2\vartheta)\right)_{ll} - Cm\left(\vartheta_l Cos^2\vartheta\right)_l -$$

$$-\left(\Lambda\vartheta_l\right)_l - \beta\vartheta_l\Lambda_l. \tag{1.16}$$

Here the dimensionless variables are introduced, scaling to the length with L ($2L$ is the length of the filament) and to the time with $\zeta L^4 / C$. The magnetoelastic number, characterizing the ratio of magnetic and elastic forces, is introduced according to the relation

$$Cm = \frac{2\pi^2 a^2\chi^2 H_0^2}{\mu + 1}\frac{L^2}{C}. \tag{1.17}$$

A set of boundary conditions necessary to solve the system of equations (15 - 16) follows from the absence of stresses and momentum stresses at the rod free ends that yields

$$\vartheta_{ll} = -CmSin(2\vartheta), \tag{1.18}$$

$$\vartheta_l = 0, \tag{1.19}$$

$$\Lambda = -CmCos^2\vartheta \tag{1.20}$$

at $l = \pm 1$.

2. Numerical simulation results

Results given by current physical model of flexible magnetic swimmer was observed in [13-14]. It was shown that there exists stabile 'U'-type configurations of flexible magnetic filaments in rotating magnetic field Fig. 1. By increasing frequency of rotation of magnetic field symmetrical configuration of filament deforms. This is essential for developing flexible magnetic swimmer inspected in current paper. If frequency of rotating magnetic field is too high there is no more chance to exist 'U' – type configurations of filament. Transition from 'U' – type configuration to 'S' – type configuration is shown in Fig. 2.

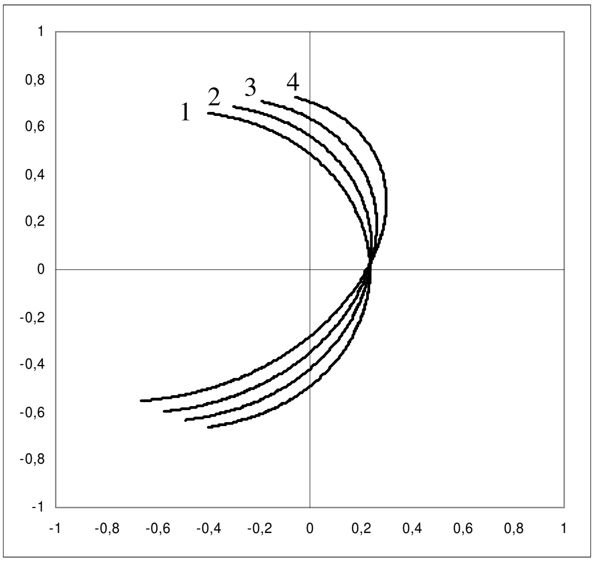

Fig. 1. Configurations of flexible magnetic filaments [14]. Magnetic field rotates anticlockwise. Coordinate system connected with magnetic field (magnetic field strength is along x axis). *1 –* ω *= 0, 2 –* ω *= 5, 3 –* ω *= 10, 4 –* ω *= 15. Cm = 50,* γ *= 0,* β *= 2.*

Let us calculate critical frequencies for existing stabile 'U'- type configurations and 'S' – type configurations of filament in rotating magnetic field. Results are shown in Fig.3. Critical frequency of magnetic field ω_c in case of 'U' – type configurations of filaments is small compared with corresponding frequency in case of 'S' – type configurations of flexible magnetic filaments. Obtained frequencies are essential to create flexible magnetic swimmer.

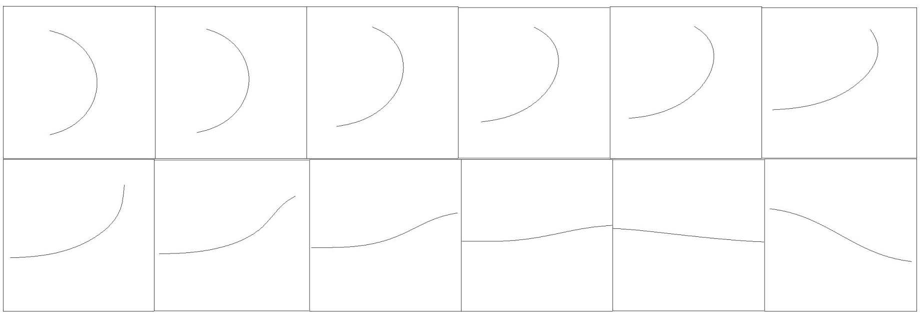

Fig. 2. Configurations of flexible magnetic filament. Magnetic field rotates anticlockwise. Coordinate system connected with magnetic field (magnetic field strength is along x axis). Starting from top left to bottom right - τ = *0*, *0.5E-5*, *1.9E-5*, *2.9E-5*, *3.6E-5*, *5.1E-5*, *5.6E-5*, *5.9E-5*, *6.4E-5*, *6.9E-5*, *8.0E-5*, *11.7E-5*. *Cm = 25,* γ *= 0,* β *= 2,* ω *= 50.*

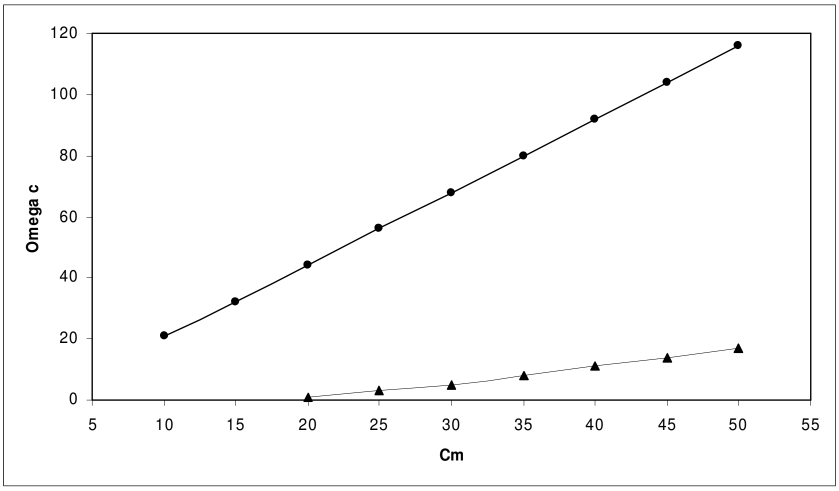

Fig. 3. Critical frequency of rotating magnetic field in dependence of magnetoelastic number *Cm*. Triangles – 'U' – type configurations of filament, circles – 'S' – type configurations of filament.

From here we can calculate relations for critical frequency in both cases of configurations

$$\omega_c[U] = 0.6Cm - 13, \tag{2.1}$$

$$\omega_c[S] = 2.4Cm - 4. \tag{2.2}$$

We can obtain critical values of magnetoelastic number *Cm*

$$Cm_c[U] = 21.7, \tag{2.3}$$

$$Cm_c[S] = 1.7. \tag{2.4}$$

Critical value of magnetoelastic number *Cm* (2.3) means that if magnetoelastic number defined in (1.17) is less then critical value (2.3) there is no possibility to create stabile 'U' - type configuration in rotating magnetic field.

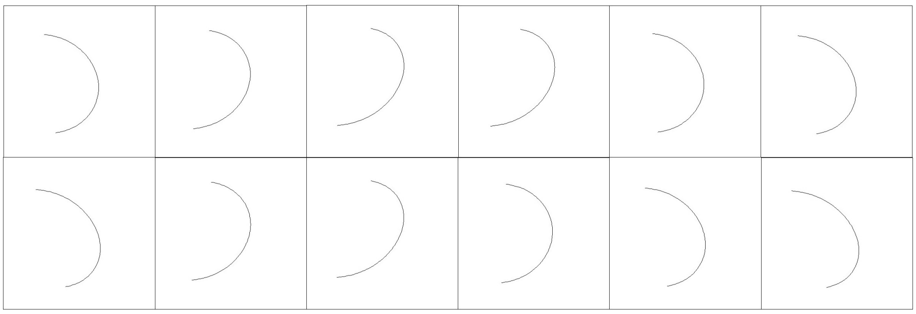

Fig. 4. Configurations of flexible magnetic filament. Coordinate system connected with magnetic field (magnetic field strength is along x axis). Starting from top left to bottom right - $\tau = 0.1820$, 0.1828, 0.1836, 0.1844, 0.1856, 0.1860, 0.1876, 0.1892, 0.1900, 0.1916, 0.1924 and 0.1932. $Cm = 50$, $\gamma = 0$, $\beta = 2$, $\omega_0 = 50$, $f_0 = 10000$.

Let us calculate dynamics of bent flexible magnetic filament in oscillating magnetic field. Bent flexible magnetic filament in case shown in Fig.4. swim in direction along to direction of strength of magnetic field with constant velocity [13-14]. In direction tangential to direction of magnetic field mass center oscillates around initial state with small amplitude about 10^{-4} of length of filament. In case of 'S' – type configuration of flexible magnetic filament motion of mass center is not observed. Obtained results differ from results given in [11]. In mentioned paper filament swims in opposite direction. In case showed in Fig.4. each half of bent filament can be described as cargo for other half of filament. Then we can imagine bent filament as half of filament with cargo in right end of filament in case shown in Fig.4. In this case we can look in nature for similar swimmers, for example, spermatozoid. As we see, direction of swimming is identical to direction obtained in current paper.

3. Conclusions

Relations describing physical properties of dynamics of flexible magnetic swimmer are obtained. Flexible magnetic filament offers a possibility to create a propulsion force in a viscous medium using oscillating magnetic field. It is shown that flexible magnetic filament repeats phenomena observed in nature. Such a filaments gives a new possibilities to the micromanipulations. The additional investigations in theoretical field will be done to describe magnetic filaments in future.

Acknowledgment

This work was supported by the European Social Fund (ESF).

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