International Scientific Colloquium
**Modelling for Material Processing**
Riga, June 8-9, 2006

# Continual Surface Induction Hardening of Profile Prismatic Steel Bodies

## P. Karban, M. Mach, I. Doležel, J. Barglik

### Abstract

The paper deals with simulation of continual surface induction hardening of 3D steel bodies that represents, from the physical point of view, a relatively complicated coupled problem. Derived is its basic mathematical model consisting of two partial differential equations with time variable boundary conditions that describe the distribution of electromagnetic and temperature fields. These fields are solved numerically by combination of professional codes and procedures developed and written by the authors. The methodology is illustrated on an example whose results are discussed.

### Introduction

Continual surface induction hardening of steel bodies belongs to modern and environmentally friendly heat treatment technologies. From the physical and mathematical viewpoints, however, its modeling is still a challenge because its represents a multiply coupled problem. The physical phenomena include magnetic field, production of the Joule losses in the processed object, its heating and consequent cooling and accompanying metallurgical changes in its internal structure. Papers that are aimed at their simulation are still rare [1]–[3]. A slightly simplified mathematical model that we use consists of the Helmholtz equation describing electromagnetic field and Fourier-Kirchhoff for nonstationary temperature field solved in the hard-coupled formulation. The model is, moreover, complicated by the time-varying boundary conditions that have to be determined for every time step.

### 1. Formulation of the problem

Consider an arrangement depicted in Fig. 1. The hardened body is a profile workpiece sufficiently long in the direction of axis $z$. It is necessary to harden the indicated surface. The inductor formed by a massive conductor moves at a slow velocity from the front part to the rear part of the workpiece. Then the heated surface is intensively cooled by the sprayer.

The aim of the paper is to propose and realize an algorithm that would allow mapping of the temperature evolution along this surface and find distribution of the final hardness.

### 2. Continual mathematical model

The mathematical model of the problem is given by two partial differential equations describing distribution of harmonic electromagnetic and nonstationary temperature fields. The fields are, moreover, characterized by time variable boundary conditions.

Because of nonlinearity of the processed workpiece the distribution of electromagnetic field is generally described by equation [4]

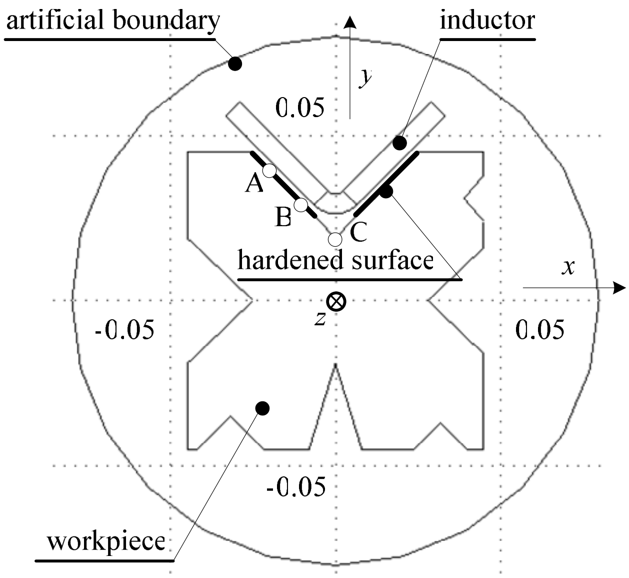


Fig. 1. The cross-section of the profile

$$\mathrm{rot}\left(\frac{1}{\mu}\mathrm{rot}\,\boldsymbol{A}\right)+\gamma\frac{\partial\boldsymbol{A}}{\partial t}=\boldsymbol{J}_{\mathrm{ext}} \tag{1}$$

where $\boldsymbol{A}$ denotes vector potential, $\gamma$ the electrical conductivity, $\mu$ the magnetic permeability and $\boldsymbol{J}_{\mathrm{ext}}$ density of the external currents. Each domain of the system (workpiece, inductor, sprayer etc.) is characterized by specific values of $\gamma$ and $\mu$.

The movement of the inductor is very slow (about 1–2 mm/s), while frequency of the field current is usually several tens of kHz. That is why it is not necessary to consider eddy currents in the workpiece induced by velocity. On the other hand, however, it is impossible to solve the problem as a transient (because of unacceptable computing time). One reason is the mentioned frequency and another reason is permeability of the workpiece. As known, it is not a function of only magnetic field, but also of temperature. And in the case of induction hardening we must work with austenitizing temperatures that exceed the Curie point. As we were not able to use equation (1), we had to develop somewhat simplified model, in which we consider only two values of permeability, each of them being constant (Fig. 2).

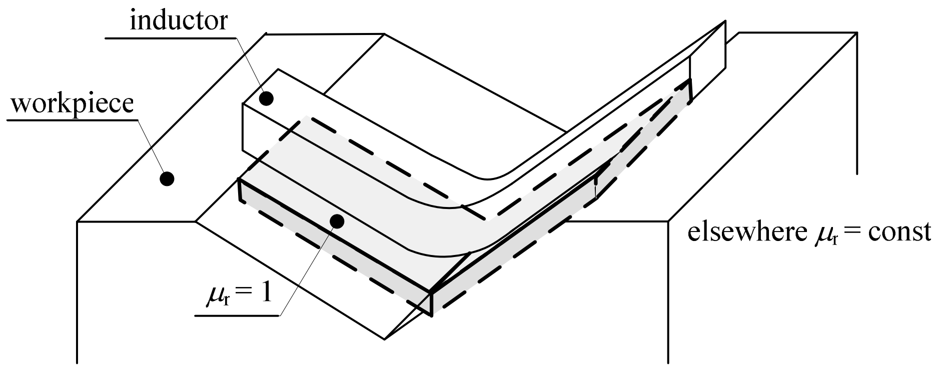


Fig. 2: Distribution of permeability in the workpiece

The value of relative permeability in a thin layer below the inductor is equal to 1 (the temperature of this part reaches or exceeds the Curie point) while in other parts it is considered constant. Dimensions of the layer below the inductor and value of the constant may be determined from testing computations providing approximate distribution of temperature and magnetic flux density in the workpiece.

On these assumption (1) may be replaced by the Helmholtz equation for the phasor values of corresponding quantities in the form

$$\mathrm{rot\,rot}\,\underline{\boldsymbol{A}} + \mathrm{j}\cdot\omega\gamma\mu\underline{\boldsymbol{A}} = \mu\underline{\boldsymbol{J}}_{\mathrm{ext}} \tag{2}$$

Now both values $\gamma$ and $\mu$ are functions of only the temperature. The definition area must be bounded by an artificial boundary placed at a sufficient distance from the system inductor-workpiece. The boundary is characterized by the Dirichlet condition $\underline{\boldsymbol{A}} = 0$.

The definition area for nonstationary temperature field is represented by the whole workpiece and its distribution is given by the Fourier-Kirchhoff equation [5]

$$\mathrm{div}\left(\lambda\cdot\mathrm{grad}\,T\right) = \rho c\cdot\frac{\partial T}{\partial t} - w_{\mathrm{Ja}}, \tag{3}$$

where $\lambda$ denotes the thermal conductivity of the material, $\rho$ its specific mass, $c$ its specific heat (all previous coefficients being nonlinear functions of temperature) and $w_{\mathrm{Ja}}$ the specific average Joule losses in the material given as $w_{\mathrm{Ja}} = \gamma\omega^2\left|\underline{\boldsymbol{A}}\right|^2$. The boundary conditions were set up according to the physical nature of the task. Generally, the heat losses in the system are caused by convection and radiation. The condition of convection of heat to ambient air is fully respected. Problems are, however, with radiation. As mentioned above, the heated part of the surface is immediately cooled by spraying water. This means that losses due to radiation are produced only for a short time of heating (several seconds). That is why we neglected them at this stage of research.

But anyway, the temperature computations require respecting of changes in boundary conditions brought about by the time evolution of heating itself and also by consequent intensive cooling. These changes are described by time-dependent distribution of the coefficient $\alpha$ characterizing the convective heat exchange. As its theoretical investigation is still extremely difficult, it was determined by an indirect approximate theoretically-empirical method based on experimental results [6].

The surface hardness is a function of the velocity of cooling, as follows from the unbalanced diagram of the considered steel in Fig. 3. The higher velocity of cooling (and shorter time), the higher value of final hardness we obtain.

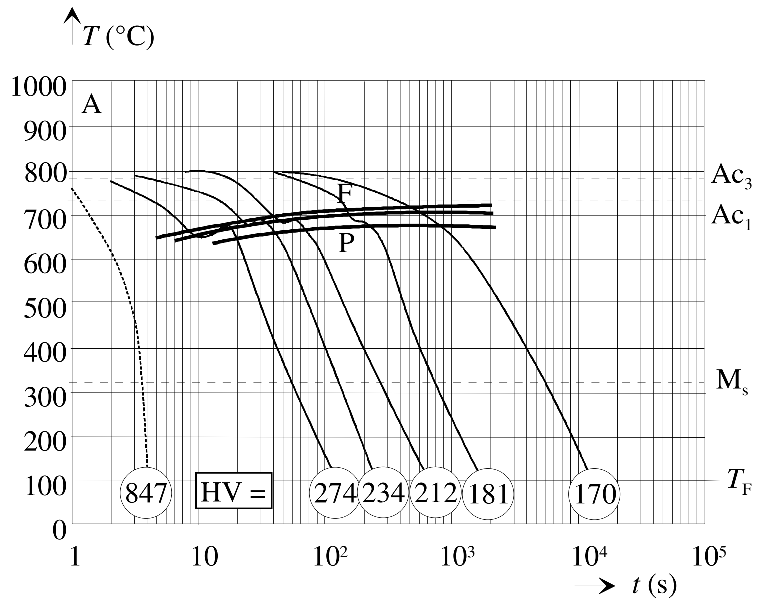


Fig. 3: The unbalanced diagram of Polish steel 50 C50U

The hardness in Vickers as a function of the time of cooling for this steel is depicted in Fig. 4.

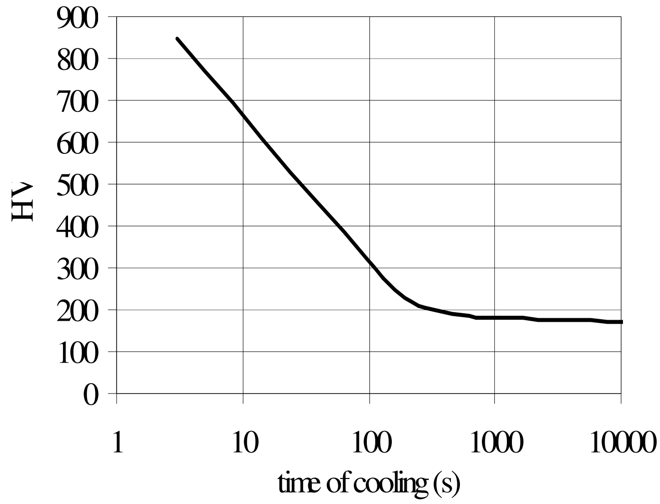


Fig. 4: Hardness of steel 50 C50U as a function of the time of cooling

## 3. Results and their discussion

The task was solved for the following parameters:

- Workpiece: Polish carbon steel 50 C50U whose unbalanced diagram is depicted in Fig. 3. Its main parameters concerning induction hardening are: $Ac_1 = 730$ °C, $Ac_3 = 780$ °C, $M_s = 325$ °C. Known are its dependencies $\gamma(T)$, $\lambda(T)$ and $\rho c(T)$. The axial length of the workpiece is $0.118$ m (coordinate $z$ of the front being $-0.059$ m, of the back $0.059$ m).
- Inductor: massive conductor of rectangular cross-section $3\times 9\cdot 10^{-3}$ m that carries current of density $J = 1.05\cdot 10^{7}$ A/m$^2$ and frequency $f = 20$ kHz. Its velocity is $0.002$ m/s.
- Convective heat transfer coefficient $\alpha = 20$ W/m$^2$K for air, $500$ W/m$^2$K for water spray (found using the method described in [6]).
- Ambient air: $T_{ext} = 30$ °C.

Computations were realized by professional code FEMLAB cooperating with a lot of user procedures developed and written by the authors.

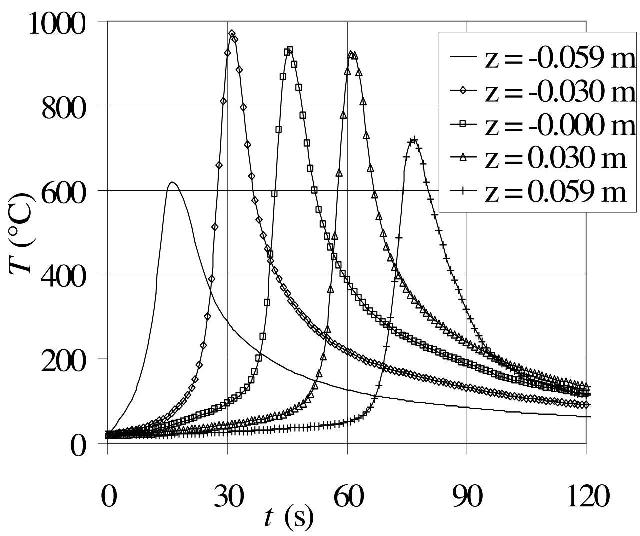


Fig. 5: Time evolution of temperature at selected point of the line passing through point $A \equiv (-0.02,\ 0.0398)$, see Fig. 1

The most important results are shown in Figs. 5–7 that depict the time evolution of temperature at selected point of lines passing through points A, B and C (see Fig. 1) parallel with axis $z$ provided that the process starts at the front of the body ($z = -0.059$ m).

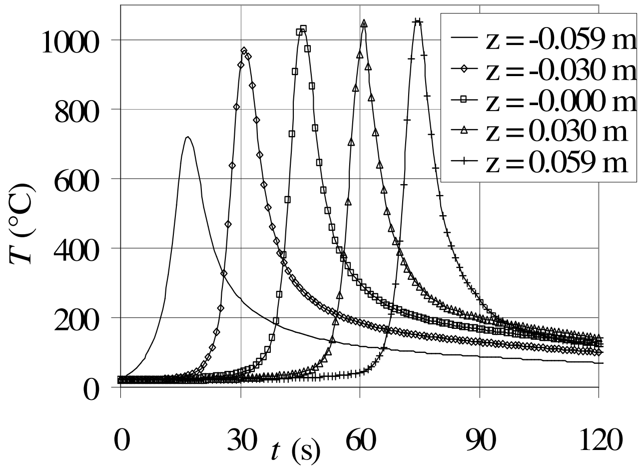


Fig. 6: Time evolution of temperature at selected point of the line passing through point $B \equiv (-0.01, 0.0294)$, see Fig. 1

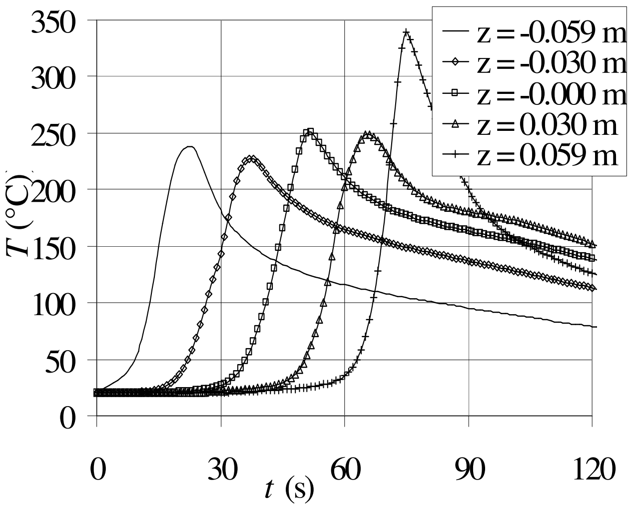


Fig. 7: Time evolution of temperature at selected point of the line passing through point C $C \equiv (0, 0.019)$, see Fig. 1

It is clear that points on line C will also be heated, but their maximum temperature is, unlike points on lines A and B, far below $Ac_3$. These points remain, therefore, unhardened, as was required according to Fig. 1.

The resultant hardness at particular points of lines A and B determined from the time of cooling to the martensite temperature is in Tab. 1.

Tab. 1: Final hardness of the body at the selected point of the surface

| line A | $z = -0.059$ m | $z = -0.03$ m | $z = 0$ m | $z = 0.03$ m | $z = 0.059$ m |
|---|---|---|---|---|---|
| hardness (HV) | – | 570 | 400 | 380 | – |
| line B | $z = -0.059$ m | $z = -0.03$ m | $z = 0$ m | $z = 0.03$ m | $z = 0.059$ m |
| hardness (HV) | – | 640 | 600 | 620 | 650 |

The highest values of surface hardness are obtained along line B. The reason is good local concentration of magnetic field and eddy currents in this region and high value of coefficient $\alpha$. As the line A is near the "upper corner" of the body, concentration of magnetic field and eddy currents is there somewhat lower, the average value of $\alpha$ as well, which results in lower values of hardness. Nevertheless, the values well correspond with experimental data (between 400–600 HV). Problems with hardness at the front and rear ends of the body (low maximum temperature) follow from the imperfectness of the algorithm used. While we considered permanent movement of the inductor with the sprayer, the real situation requires that both at the beginning and end of the process the inductor must stop for a while in order to produce necessary temperature in the body.

## Conclusions

Mathematical and computer simulation of phenomena associated with induction hardening is a very complicated business. Beside the fact that the basic mathematical model is described by two nonlinear and nonstationary partial differential equations for electromagnetic and temperature fields with time-dependent boundary conditions, we must work with a lot of input data that have to be obtained experimentally (for example parameters of continual cooling of heated part by water sprayer). That is why it is necessary to find, particularly for complex 3D arrangements, certain simplifications that still provide physically acceptable results.

One of the simplified models is presented in this work. But, despite of neglecting seemingly important items (replacement of nonlinear material by linear one by parts with a step change of permeability at the Curie point etc.) the results are still not far from the physical reality, as was validated by experiments.

Next work in the field will be aimed at further improvement and acceleration of the used algorithms and higher accuracy of results.


## Acknowledgment

This work was financially supported by the Czech Ministry of Education (Research Plan VZ MSM 4977751310) and Polish Scientific Research Committee (Grant Project 3T08C 01630) and realized within the framework of the multigovernmental collaboration between Poland and the Czech Republic.


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## Authors


Ing. Karban, Pavel
Ing. Mach, Martin
Faculty of Electrical Engineering
University of West Bohemia
Univerzitní 26
306 14 Plzeň
Czech Republic
E-mail: karban, mach@kte.zcu.cz

Prof. Dolezel, Ivo
Institute of Thermomechanics
Czech Academy of Sciences
Dolejškova 5
182 02 Praha 8
Czech Republic
E-mail: dolezel@iee.cas.cz

Prof. Barglik, Jerzy
Department of Electrotechology
Silesian University of Technology
Krasinskiego 8
40-019 Katowice
Poland
E-mail: jerzy.barglik@polsl.pl