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Heat and Mass Transfer in a Cylindrical Vessel with the Melt Exposed to the Influence of Combined Electromagnetic Fields

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Abstract

The current paper contributes to the study of the characteristics of heat/mass transfer in the melt in a cylindrical vessel under the influence of previously not considered superpositions of pulsating and steady, pulsating and rotating electromagnetic fields. The obtained results testify to a possibility to significantly expand the range for controlling the heat/mass transfer characteristics under such combined influence.

Introduction

The control of hydrodynamics and heat/mass transfer in different technologies, where electrically conducting liquids are either the object of production (for example, in metallurgy) or a functional working medium (for example, in liquid metal systems of aggregates for nuclear engineering) is an acute problem presently as well. To decide this problem, the most promising are likely magnetohydrodynamic (MHD) methods of influence on electrically conducting media (molten metals, melts of semiconductors, melts of salts, etc.) by electromagnetic field of different types (steady and alternating [1, 2]).

Yet, in many cases the use of any magnetic field either is ineffective or accompanied by some unwanted from the practical point of view effects. That is why a new trend in MHD – investigation of possibilities to control hydrodynamics and heat/mass transfer by different combination of electromagnetic (steady and alternating) fields – is having been developed for the recent 10 years. The investigations are carried out in two directions: elaboration of methods to solve problems under simultaneous action of superposition of different fields on the liquid and analysis of possibilities to unite magnetic fields of different types practically in one device.

The obtained so far results on the influence of combined magnetic field on the transfer phenomena are presented in [1-3]. With reference to the classification of possible versions of field superpositions given in [2], unstudied still remain the superpositions of pulsating and rotating and pulsating and steady magnetic fields. The present paper is concerned with the investigation of the above fields' influence on the hydrodynamics and heat/mass transfer in molten metals. Methods of numerical simulation of electrodynamic, hydrodynamic, heat and concentration problems formulated for the considered conditions have been applied.

1. Method for Solution

The solution of the general problem on the interaction of different combinations of electromagnetic fields with the conducting liquid is described in detail in [3]. As a rule, the calculation includes three basic parts:

- a calculation of the distribution of external mass forces affecting the conducting liquid;

- a calculation of the melt flow velocity pattern driven by the external forces;

- a calculation of the distribution of temperature and dopant concentrations at forced convective flows in the melt driven by electromagnetic forces.

In all situations, a cylindrical volume of radius R_0, height h, and electric conductivity σ, filled with a melt, is considered. This cylindrical volume is placed co-axially into a cylindrical inductor of radius R and height h_0, which induces a combination of alternating or an alternating and a steady magnetic fields. The magnetic fields can have all three components. The liquid cylindrical volume is characterized by a density ρ, a specific electric conductivity σ and a kinematic viscosity ν.

Omitting the details of the solution procedure of the general problem [3-5], we mention only the basic definitions of some characteristic values for the situations under discussion. To calculate forces induced by an alternating electromagnetic field, a set of Maxwell equations for a quasi-stationary field was solved:

$$\mathbf{rotH} = \mathbf{j}; \qquad \mathbf{rotE} = -\frac{\partial \mathbf{B}}{\partial t} = -i\mu_0\omega\mathbf{H}, \tag{1}$$

which, by excluding the functions **H** or **E,** is related to a Helmholtz equation (Eq.2), if the field is considered inside the conductor, or to a Laplace equation for the field in a non-conducting medium:

$$\Delta\mathbf{A} = i\sigma\mu_0\omega\mathbf{A}, \qquad \Delta\mathbf{A} = 0. \tag{2}$$

Here, **A** denotes any of vectors **H** or **E**, μ_0 is the magnetic permeability of vacuum, ω is the angular velocity of the current, σ is the specific electric conductivity of a medium.

The obtained numerically values of the fields **H** and **E** and, respectively, the values of the electric current **j** and magnetic induction **B** are used for the definition of the average in time density of the electromagnetic force **f**, affecting the liquid, from the formula:

$$\mathbf{f} = \frac{1}{2}\Re e\left[\mathbf{j_0} \times \mathbf{B_0^*}\right]. \tag{3}$$

To calculate the density distribution of forces induced by the interaction of a moving electrically conducting liquid and an external steady magnetic field (in an induction-free approximation), the following expression is used:

$$\mathbf{f} = \sigma(\mathbf{v} \times \mathbf{B}) \times \mathbf{B}. \tag{4}$$

To calculate the distribution of buoyancy forces, occurring in a non-uniformly heated liquid due to a difference in its density from different temperatures, the following expression is used:

$$\mathbf{f}_{zA} = \rho\beta g(T - T_0)\mathbf{e}_z, \tag{5}$$

where β is the coefficient of volumetric expansion of the liquid, g is the free fall acceleration, $(T - T_0) = \Delta T$ denotes a temperature drop along the cylinder vessel height.

Velocities in the liquid volume, occurring due to actions of different mass forces, were calculated by solving the set of Navier-Stokes equations, where the above three types of forces were used as external.

To simplify the calculation procedure, in a general case, other functions are substituted into the Navier-Stokes equations instead of velocity and pressure: an angular velocity of liquid rotation $\Omega = v_\varphi/r$ and the moment of the azimuthal component of the rotor velocity $W = r(\partial v_z/\partial r - \partial v_r/\partial z)$. As a result, we have a set of four equations, which in the dimensionless forms reads [3]:

$$r^2 \frac{\partial \Omega}{\partial t} = \mathrm{div}\left[r^2\left(\nu \,\mathbf{grad}\,\Omega - \mathbf{v}\Omega\right)\right] + \sum_{i=1}^{k} F_{1i}\,, \tag{6}$$

$$\frac{1}{r^2}\frac{\partial W}{\partial t} = \mathrm{div}\left[r^{-2}\left(\nu \,\mathbf{grad}\;W - \mathbf{v}W\right)\right] + \frac{\partial(\Omega^2)}{\partial z} + \sum_{i=2}^{k} F_{2i}\,, \tag{7}$$

$$\mathbf{v} = \frac{\mathbf{e}_\varphi}{r}\times \mathbf{grad}\;\Psi\,, \qquad W r^{-2} = \mathrm{div}\left[r^{-2}\mathbf{grad}\;\Psi\right]. \tag{8, 9}$$

The right terms in Eqs.(6,7) characterize the value of external forces. F_1 in Eq.(6)characterizes the azimuthal force component, and F_2 in Eq.(7) – the azimuthal component of rotor force.

In case there is no azimuthal velocity component in the volume, for example, when the melt is affected by only a travelling or a pulsating electromagnetic field, then $\Omega = 0$, and only Eqs.(7-9) remain from the set of Eqs.(6-9).

As typical values, we have chosen: for time t - ν/R_0^2, for Ω - R_0^2/ν; for W - R_0/ν and for $\Psi = 1/(\nu R_0)$. All values having length are related to R_0 – the radius of the conducting cylinder.

The temperature field within the conducting liquid volume subjected to different combinations of electromagnetic fields was calculated by solving the equations of convective heat transfer as

$$\frac{\partial T}{\partial t} = \mathrm{div}\left[-T\cdot v + \frac{1}{\mathrm{Pr}}\cdot \mathbf{grad}T\right], \tag{10}$$

where Pr is the Prandtl number.

Since the temperature distribution is a significant factor, which affects the melt motion, the equation of heat transfer (10) has been solved in a joint set together with the equations of hydrodynamics.

To define the concentration distribution within the liquid volume, the equation of mass transfer reads:

$$\frac{\partial c}{\partial t} = \mathrm{div}\left[-cv + \frac{1}{\mathrm{Sc}}\mathbf{grad}c\right], \tag{11}$$

where Sc is the Schmidt number.

The boundary conditions are given by the problem under solution, in particular, for the stream function ψ - the symmetry conditions on the axis and the conditions of sticking on the wall:

$$\psi_{r=0} = 0, \quad \psi_w = 0,$$

for Ω: $$\Omega_w = 0, \quad \left.\frac{\partial \Omega}{\partial r}\right|_{r=0} = 0, \quad \text{for } W\big|_{r=0} = 0, \quad W\big|_w = \frac{2(\psi_{w+1} - \psi_2)}{2h^2} + 0\,(h).$$

For the heat/mass transfer equations, the boundary conditions are given with respect to the requirements of the problem under solution. For the below situations, the problem was solved for a linear distribution of temperature on the side surface of the cylinder and under condition that at the initial time the dopant concentration in the vessel was uniform:

$$T\big|_{r=R} = f(z), \quad T\big|_{z=0} = T_c, \quad \frac{\partial T}{\partial r}\bigg|_{r=0} = 0, \quad T\big|_{z=l} = T_c,$$

$$-\frac{\partial c}{\partial z}\bigg|_{z=0} + Sc(1-k)c = 0, \quad \frac{\partial c}{\partial r}\bigg|_{r=R} = 0,$$

$$c\big|_{z=l} = 1, \quad \frac{\partial c}{\partial r}\bigg|_{r=0} = 0, \quad c(r,z,t)\big|_{t=0} = f(r,z).$$

2. Calculation Results

Figs.1 – 4 illustrate the data obtained for the superimposed rotating and pulsating magnetic fields (RMF + PMF). Fig.1 illustrates the distributions of the f_r, f_φ and f_z-components of the force, affecting the liquid.

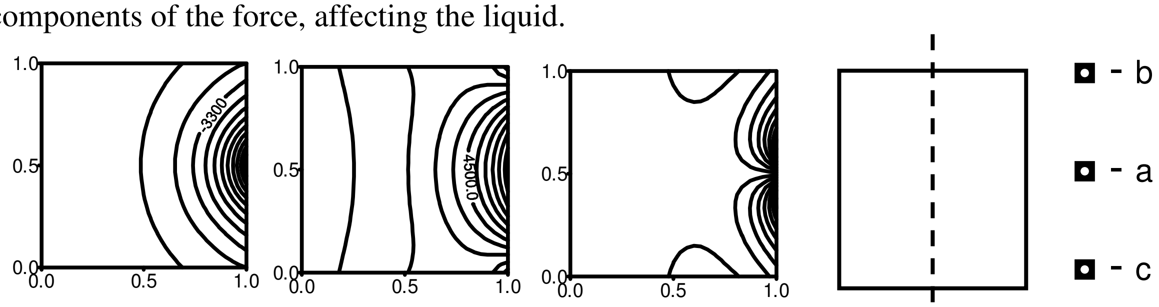

Fig. 1. Distributions of the f_r, f_φ and f_z-components of the force and the locations of the PMF inductor. ◘ - locations of the PMF inductor.

Fig.2 displays the stream functions for situations when the PMF acts in the volume center (Fig.2a), at the top (Fig.2b), and at the volume bottom (Fig.2c). Typical shapes of isotherms for the above situations are shown in Fig.2d.

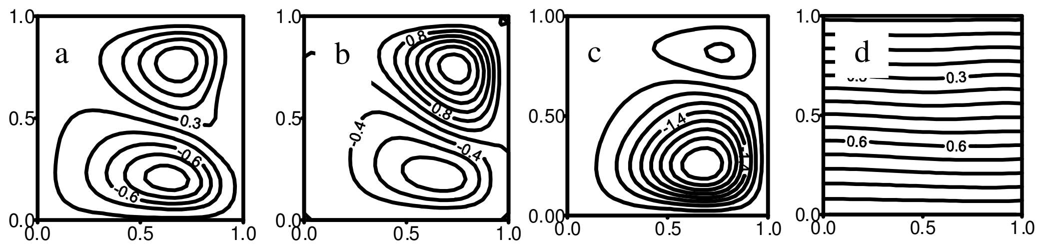

Fig. 2. The stream functions for situations when the PMF acts in the volume center (a), at the top (b), and at the volume bottom (c) and typical shapes of isotherms (d).

Fig.3 illustrates similar results obtained at a larger Grashof number (Gr). Fig.4 shows distributions of concentrations along the height and radius of the volume. The diagrams show that the forming velocity patterns, the distributions of temperatures and concentrations are determined by the relation of the intensity of the acting field and the thermogravity flows.

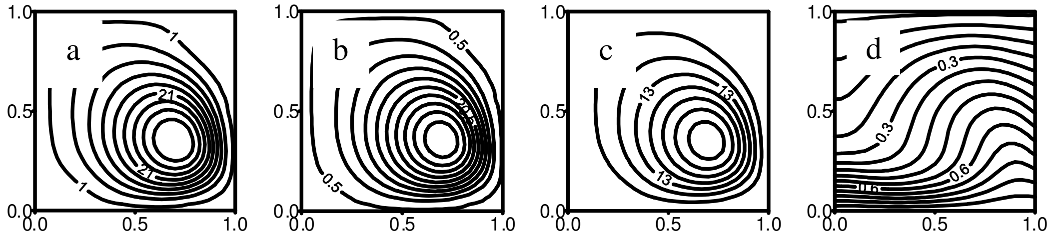

Fig. 3. The stream functions for situations when the PMF acts in the volume center (a), at the top (b), and at the volume bottom (c) and typical shapes of isotherms (d).

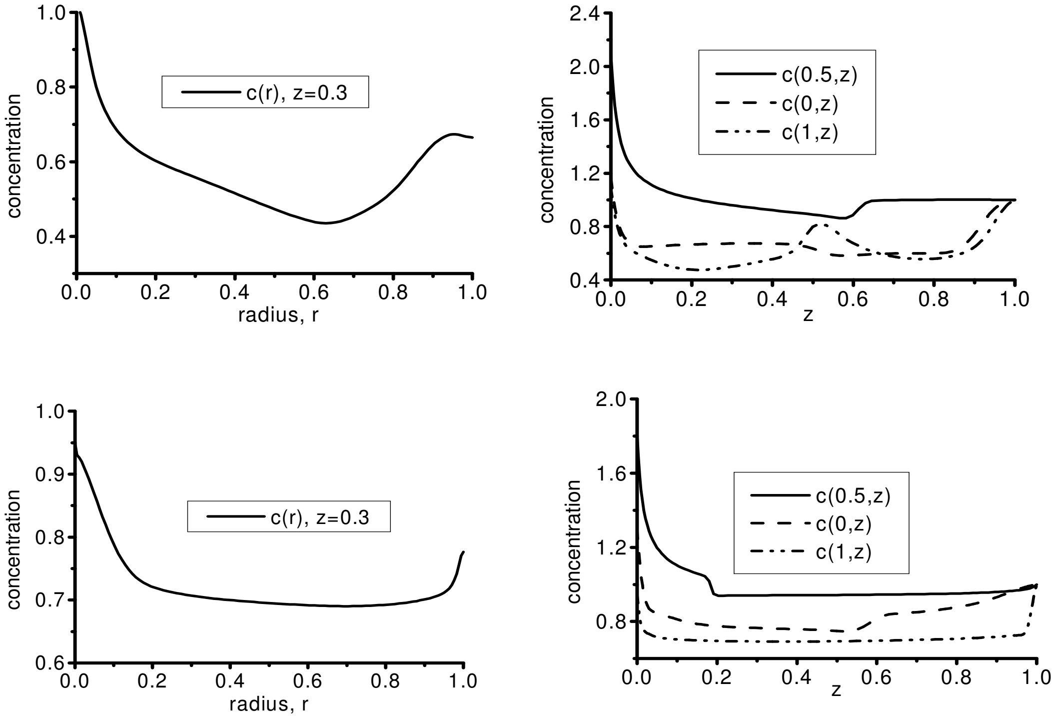

Fig. 4. Distributions of concentrations along the radius and height of the volume.

Figs.5 – 6 depict the typical numerical dependencies for stream functions, temperature and concentration distributions for the superimposed pulsating and steady magnetic fields (PMF + SMF). Fig.5a,b,c illustrate the results obtained at different positions of a PMF inductor along the vessel's height. Fig.5d presents typical isotherm dependencies for the above situations.
Fig.6 show typical curves, illustrating concentration distributions. Similarly to the above case, the run of dependencies is determined by a quantitative relation of thermogravity and electromagnetic forces and can produce various results. In every definite case the intensity of action of each force type must be accounted for.

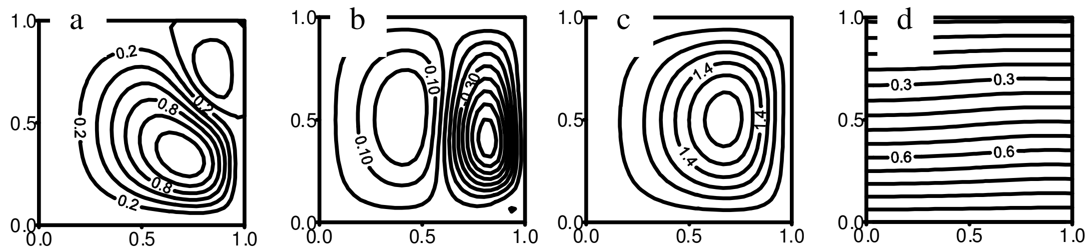

Fig. 5. The stream functions for situations when the PMF acts in the volume center (a), at the bottom (b), and at the volume top (c) and typical shapes of isotherms (d).

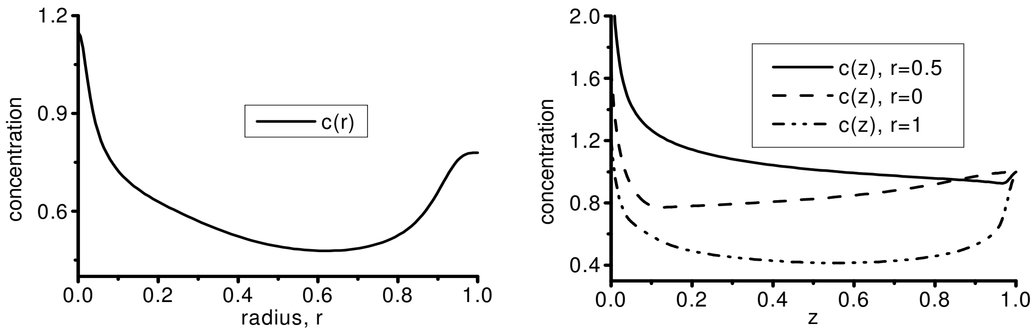

Fig. 6. Distributions of concentrations along the radius and height of the volume.

Conclusion

The obtained results on the influence by combinations of different magnetic fields on the transfer processes in molten melts do not principally differ from the results obtained in previous studies [1-3]. They have proved again the conclusion that the imposition of combined magnetic fields significantly expands the possibilities for controlling the transfer phenomena in the liquid.

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