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Flow through Porous Media at Moderate Reynolds Number

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Abstract

In modelling of flow through porous media inertia-effects must sometimes be considered. This is often done by usage of the empirically derived Ergun equation that can describe the response of several porous media but does not reveal the real mechanisms for the flow. In order to increase the understanding of such flows we have therefore performed a micromechanically based study of moderate *Re'* flow between parallel cylinders using a Computational Fluid Dynamics approach. The simulations are carried out with quality and trust by using grid refinement techniques and securing that the iteration error is sufficiently small. Main results are that the Ergun equation fits well to simulated data up to *Re'* 20, that inertia-effects must be taken into account when *Re'* exceeds 10 and that results from stationary simulations replicate time resolved ones at least up to *Re'* 880.

Introduction

Fluid flow through porous media takes place in a number of technical areas including ground water flows, flow through embankment dams, paper making, composites manufacturing, filtering, drying and sintering of iron ore pellets. In several of these areas the flow can simply be described by Darcy's law, which in its general and one-dimensional form may be written as

$$u_i = \frac{K_{ij}}{\mu} p_{,j} \text{ and } \frac{K}{\mu}\frac{\Delta p}{L} = \frac{Q}{A}, \tag{1a-b}$$

respectively. In these equations u is the superficial velocity, K the permeability, μ the viscosity of the fluid, p the pressure, Q the flow rate through an area A and Δp the pressure drop over an length L in the stream-wise direction. Darcy's law is valid as long as *Re* is sufficiently low, the fluid can be treated as incompressible and Newtonian and the porous medium is fixed. When *Re* increases over a certain level, which may be the case for erosion in embankment dams and drying of iron ore pellets, the pressure drop becomes higher than what is predicted by (1a-b) and additional non-linear terms are introduced in the so called Forchheimer equation

$$\frac{K}{\mu}\frac{\Delta p}{L} = \frac{Q}{A} + b\left(\frac{Q}{A}\right)^m \tag{2}$$

where b is a property of the porous media and m is a measure of the influence of fluid inertia. The equation was later modified by Ergun by fittings to experimental data according to

$$\frac{\Delta p}{L} g = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu \frac{Q}{A}}{D_p^{\ 2}} + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho \left(\frac{Q}{A} \right)^2}{D_p} \tag{3}$$

where ε is the fractional void volume in the bed and D_p is the effective diameter of particles [1]. From (3) a modified Reynolds (*Re'*) number can be defined

$$\mathrm{Re}' = \frac{\rho D_p \frac{Q}{A}}{\mu} \frac{1}{1-\varepsilon}. \tag{4}$$

The Ergun equation does not have a micromechanical basis and the geometrical parameters shaping the form of the equation are therefore unknown. Previous works also indicate that the parameters of the Ergun equation may be improved [2, 3].

To increase the knowledge about higher *Re* flow through porous media we will here study the detailed flow through an array of quadratic packed parallel cylinders with Computational Fluid Dynamics (CFD). The CFD simulations are performed with the commercial software ANSYS CFX 10.0 with quality and trust regarding grid refinement and iterative errors. To achieve this within reasonable computational time the CFD simulations are run with a parallel computing procedure and by this retaining the iterative error low within reasonable computing time. Another effect that is studied is whether unsteady effects take place at higher *Re* and if this effect is of importance for the apparent permeability.

1. Numerical setup

In order to simplify the simulations unit-cells are defined for steady and unsteady flow, see Fig. 1. For both unit-cells, the discretization is performed with ANSYS ICEM CFD 10.0 Hexa where the block-structure is defined and projected onto the geometry to achieve as correct and smooth description of the unit-cell as possible. For the grid refinement study the grid is refined uniformly in the in-plane directions while the number of nodes in the out-of-plane direction is two for all cases. For the unsteady simulations the geometry is mirrored so the whole cylinder is encapsulated in the unit-cell. Consequently the domain is doubled and so is the number of nodes required for a certain mesh density. In order to drive the flow perpendicular to the cylinders a pressure gradient is introduced into the momentum equations by in a sub-domain specifying a general momentum source [4].

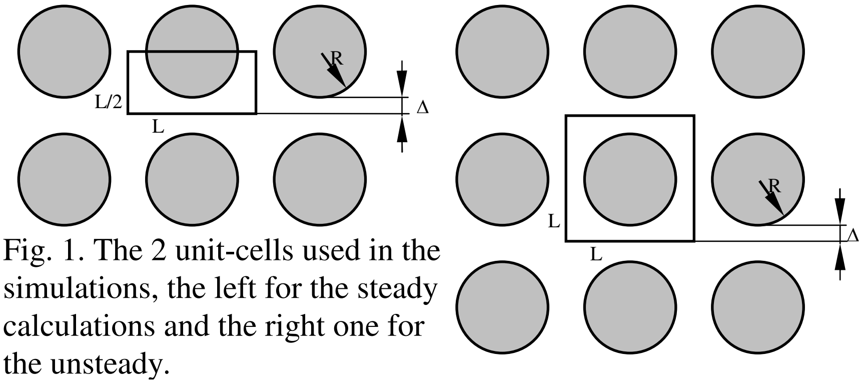

Fig. 1. The 2 unit-cells used in the simulations, the left for the steady calculations and the right one for the unsteady.

Regarding the boundary conditions for the simulations the top and the bottom of the unit-cells are defined as symmetry-planes, the cylinder wall is assumed to be smooth with a no-slip condition and the left- and right-hand sides of the unit-cells are periodic domain interfaces representing the structure of the array. The advection scheme used to solve the continuity and momentum equations are strictly 2nd order accurate, which is realised by setting the specified blend factor equal to 1 in CFX-Pre. The simulations are considered well converged when the Root Mean Square (RMS) -residuals has dropped 5-6 decades and the max-residuals is no more than 1.5 decade above the RMS-residuals.

For the simulations with *Re* less than 150 the flow-field is solved by assuming the flow to be steady. At higher *Re* both steady and unsteady simulations are carried out following the results in the literature [5, 6]. The convergence criteria are the same as for the steady simulations and the mass-flow quantity is stabilized to a constant value.

2. Results

This part is divided into 3 sections dealing with grid refinement, effect on apparent permeability from solid fraction and *Re'* and the unsteady calculations evaluated in order to check whether or not it is required to use an unsteady approach in order to get the correct apparent permeability.

2.1. Grid refinement study

The grid refinement study is performed with a solid fraction of 60% and a *Re'* of approximately 12 for 10 mesh densities, see Tab 1. The parameter of interest is the apparent permeability which is derived by usage of (1) where *Q* is calculated as the mass-flow obtained from CFX-Post divided with the density of the fluid. A polynomial fit to the results indicates that the simulations are in the asymptotic range since the curve approaches a specific value as the grid is refined, see Fig. 2. The simulated values are in all cases close to the asymptotic one with a difference of only 0.3 per mil for the mesh with 370 000 nodes. This accuracy is by all means god enough and since the computational time is short enough for this case all simulations, from now on, are performed with a mesh density in this range.

Tab.1. Meshes with the corresponding permeability.

Normalized number of nodes	Number of nodes	K
8.72	125 000	7.3839E-08
5.74	190 000	7.3825E-08
3.89	280 000	7.3815E-08
2.95	370 000	7.3810E-08
2.22	490 000	7.3807E-08
2.02	540 000	7.3805E-08
1.73	630 000	7.3803E-08
1.43	760 000	7.3801E-08
1.18	920 000	7.3800E-08
1	1 090 000	7.3798E-08

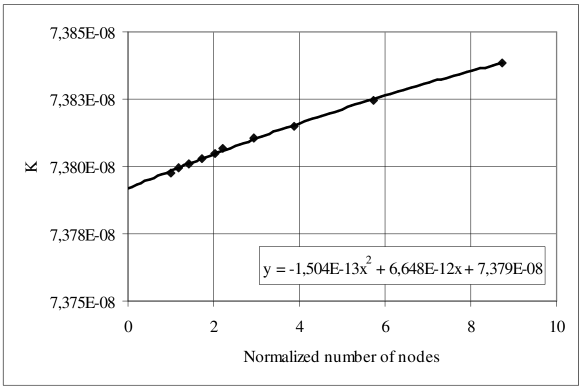

Fig. 2. Plot of the different meshes and the corresponding permeability

2.2. Different solid fractions

The simulations are performed with solid fractions of 40, 60 and 70% at a number of *Re'* ranging from about 0.001 to around 1000. At the creeping flow regime (*Re'* < 1) true permeability data are obtained see Fig. 3. As the flow-rate is increased the apparent permeability decreases for all solid fractions but predominantly for the highest one. To shed some light on this phenomenon the flow field is plotted for two solid fractions and three *Re'* Fig. 4 and 5. As seen the circulation zone and the corresponding stagnation points formed move towards the nip as *Re'* is increased. The circulation zone then becomes small and the velocity near the wall high indicating large losses.

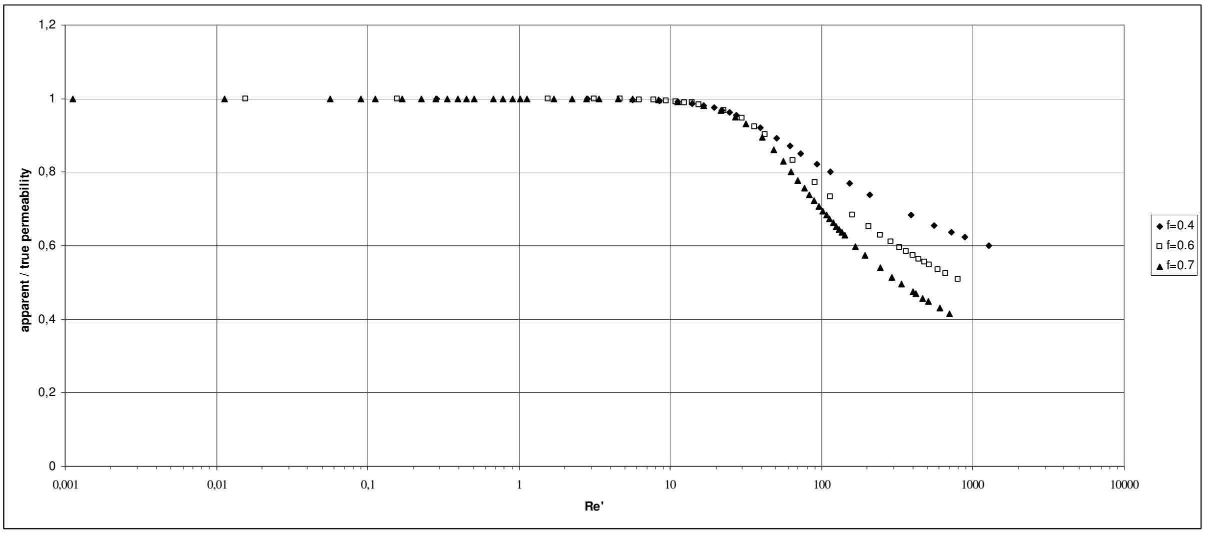

Fig. 3. The result from the simulations with different solid fractions, the apparent permeability is normalized with the true permeability and then plotted against *Re'*.

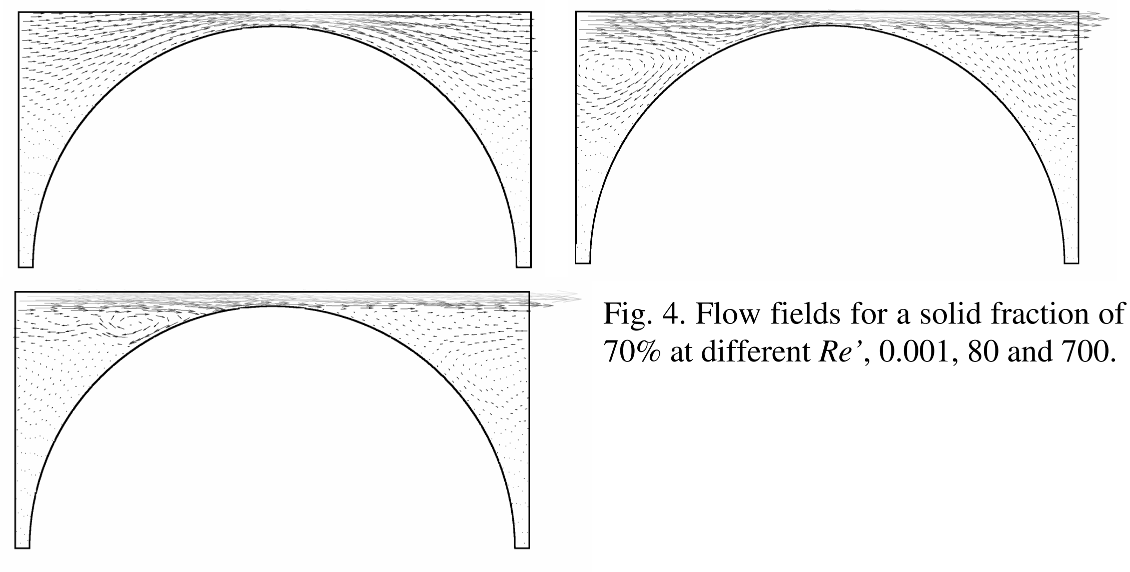

Fig. 4. Flow fields for a solid fraction of 70% at different *Re'*, 0.001, 80 and 700.

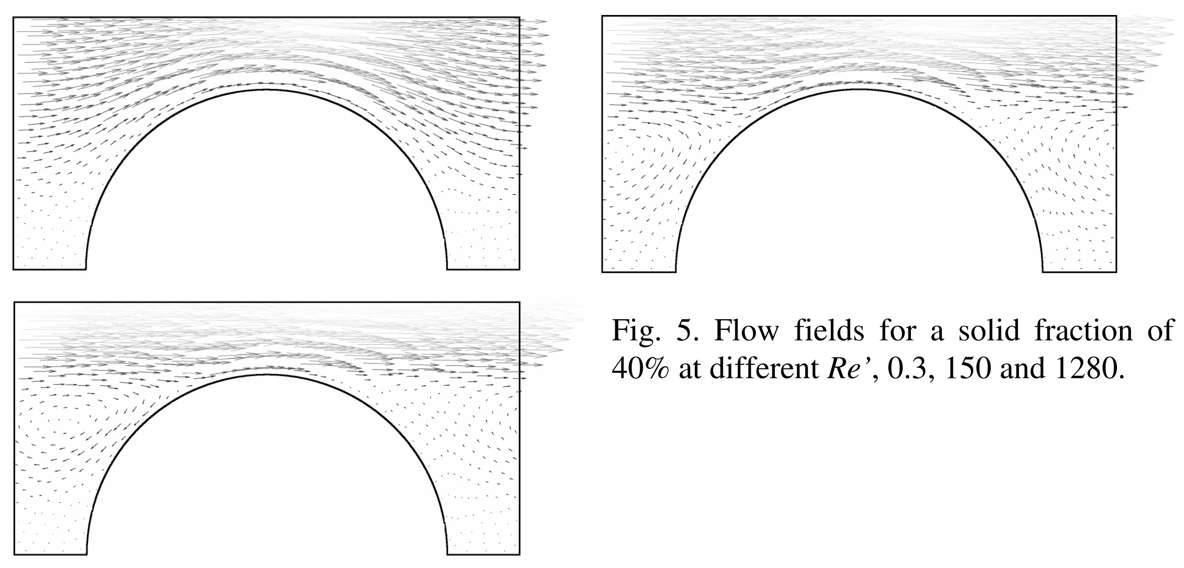

Fig. 5. Flow fields for a solid fraction of 40% at different *Re'*, 0.3, 150 and 1280.

When comparing the simulations at f = 60% with the Ergun equation it is obvious that the simulation exhibit fair agreement up to *Re'* of about 20 Fig. 6. where the parameter evaluated are the Blake-type friction factors

$$f' = \frac{\Delta p}{\rho u^2} \frac{D_p}{L} \frac{\varepsilon^3}{1-\varepsilon} \text{ and } f' = 1.75 + \frac{150}{\mathrm{Re}'} . \tag{5}$$

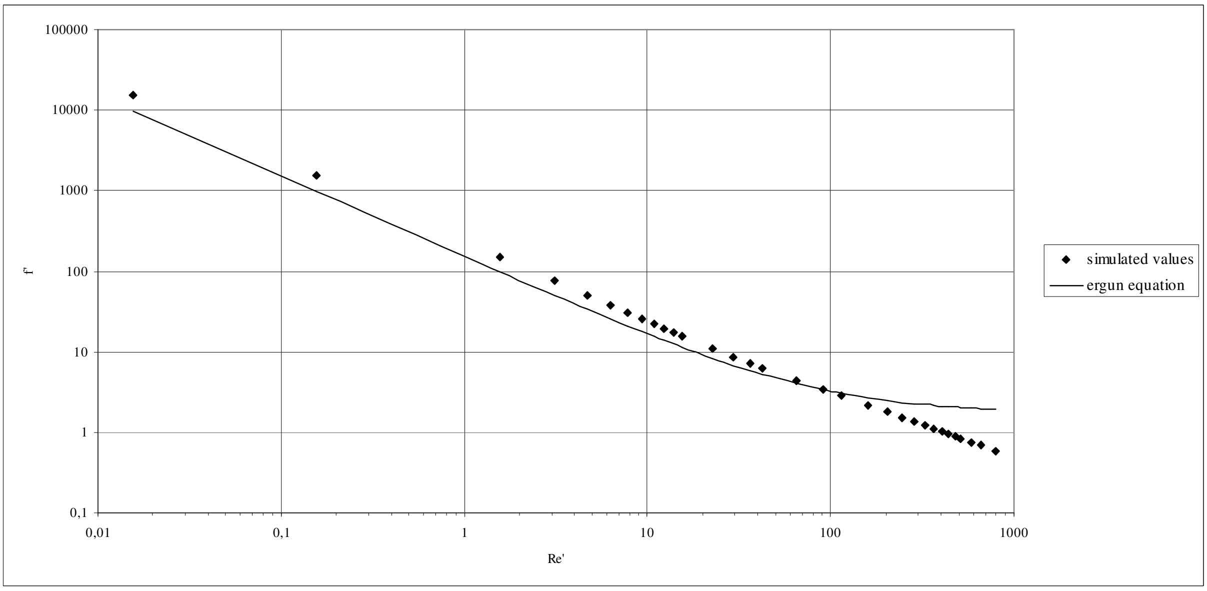

Fig. 6. Comparison between the simulated data for a solid fraction of 60% and the Ergun equation.

Above this value the slope of the Ergun curve decreases while the gradient of the simulated data stays more or less fixed. This phenomenon indicates that the empirical constants should be upgraded to suit the problem studied better.

2.3. Unsteady simulations

When comparing the steady and the unsteady simulations at *Re'* equal to 880 it appears that the final flow fields differs marginally but that the apparent permeabilities are in-principal the same. In addition the residuals for the steady calculations performed do not oscillate, indicating a steady behaviour of the flow. Unsteady calculations when *Re'* is less than 880 are therefore not required, which considerably reduces the computational time needed to achieve reliable values of the simulated apparent permeability.

Conclusions

The results from the simulations are verified with a grid refinement study and are also in the same range as results from previous work indicating that the simulations are reliable and can be trusted. This is very important since there are by definition uncertainties with numerical simulations.

The simulations show that the limit of *Re'* when inertia-effects must be taken into account when simulating flow through porous media is 10.

The Ergun equation fit well to the simulations up to *Re'* about 20, however the discrepancy thereafter shows that the need to update the constants in the Ergun equation is rather obvious.

The simulations show that steady calculations can be used at least up to *Re'* equal to 880.

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