

An Optimal Method for 3-D Numerical Simulation of Electromagnetic Induction Heating Processes

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Abstract

This paper describes an efficient computational method for 3-D numerical simulation of induction heating processes based on the current vector potential (T) and reduced magnetic scalar potential (Ψ) formulation of the electromagnetic field equations. Use of a hybrid differential-integral formulation confines the solution domain of the open boundary electromagnetic field problem to the metal, and permits full coupling of electromagnetic and heat transfer aspects of the induction heating process. The governing electromagnetic and heat transfer equations were solved using finite element method. The versatility of the developed algorithm for handling complex geometries and coil configurations is demonstrated by presenting results of numerical simulation of transverse flux induction heating (TFIH) of a steel sheet during rolling

1. Introduction

Induction heating of metals is increasingly being used in a broad range of metallurgical applications. These include preheating of billets and slabs prior to forging and rolling operations, and induction hardening of steel [1]. In all such applications, a high-frequency alternating magnetic field (of the order of kHz) is used to induce electric currents in the metal (workpiece) so as to dissipate energy as heat (Joule heating) and increase the temperature of the workpiece. The induced current is usually confined to the surface layers of the workpiece and the thickness of the induced current layer is controlled by the skin depth, which is a function of frequency, electrical conductivity, and permeability. Induction strip heating is another area, in which induction heating has been successfully used, especially, in the steel industry.

In all induction heating process, the induction coil has significant influence on the eddy current distribution; considerable effort has been expended in the last two decades on its design. In addition to this important design parameter, the magnitude of the eddy current is dependent on the frequency, RMS value of the inductor current (applied current), width, and material characteristics of the strip. In order to evaluate the influence of the different design parameters on the heating of the strip, a three-dimensional coupled eddy current and temperature model is essential.

The equations that govern the calculation of the electromagnetic field and the resultant Joule heating in such systems are well established. These equations may be represented by combining the well-known Maxwell's equations with the transient heat conduction equation. While these equations can be readily solved numerically on an individual basis, numerical simulation of coupled electromagnetic and thermal phenomena in induction heating systems presents a major challenge particularly for three-dimensional systems. The computational techniques for electromagnetic and thermal problems are generally incompatible. The electromagnetic field problem is an open boundary, one with

boundary conditions defined only at infinity. Discretization of the free space by finite element or control volume methods is cumbersome and computationally inefficient [2, 3].

Recently, a number of hybrid integral-differential methods have been proposed to compute the electromagnetic field only in the conducting domain [4, 5]. While these methods seem to be appropriate for induction heating problems, unfortunately, they are limited to two-dimensional problems. The work to be described in the present paper is in essence an extension of this approach to three-dimensional problems. The principal contribution of this work is to present a potential formulation of the electromagnetic field which limits the solution domain to the conducting region. The numerical methodology described in this paper circumvents this problem by restricting the domain of solution to the strip for both the electromagnetic and thermal problem. While the current method focuses on constant properties to elucidate the methodology, this method can be easily extended to problems with material properties dependent on temperature.

2. Governing Equations

The electromagnetic field in 3-D eddy current systems is best represented in terms of potentials to avoid discontinuity of the electric field at boundaries of the conducting domain. In this work, the current and magnetic field in the conducting domain is formulated in terms of current vector potential, \mathbf{T} ($\mathbf{J} = \nabla \times \mathbf{T}$), and reduced magnetic scalar potential, Ψ , which may be expressed as:

$$\mathbf{H} = \mathbf{T} - \nabla \Psi \quad (1)$$

where \mathbf{J} is current density, and \mathbf{H} is magnetic field intensity.

From Faraday's equation and Ohm's law, the differential equation for \mathbf{T} and Ψ may be written as:

$$\nabla^2 \mathbf{T} = -j\sigma\omega\mu_0(\mathbf{T} - \nabla \Psi) \quad (2)$$

$$\nabla^2 \Psi = 0 \quad (3)$$

where σ and μ_0 are the electrical conductivity and magnetic permeability of the metal, respectively, while ω is the frequency of the applied magnetic field.

The boundary conditions for \mathbf{T} that enforce zero current across the outer surface of the conducting domain are:

$$T_i = 0 \forall i = 1, 3 \quad (4)$$

In addition, the continuity of the normal component of the current field across boundaries requires the current vector potential to satisfy the condition that:

$$(\nabla \times \mathbf{T}_1) \cdot \mathbf{n}_1 = (\nabla \times \mathbf{T}_2) \cdot \mathbf{n}_2 \quad (5)$$

where \mathbf{n} is an outward pointing unit normal to a surface. On the other hand, the continuity of the tangential component of the electric field is given by:

$$\frac{1}{\sigma_1} \cdot (\nabla \times \mathbf{T}_1) \times \mathbf{n}_1 = \frac{1}{\sigma_2} \cdot (\nabla \times \mathbf{T}_2) \times \mathbf{n}_2 \quad (6)$$

$$\mathbf{T}_1 \cdot \mathbf{n}_1 = \mathbf{T}_2 \cdot \mathbf{n}_2 \quad (7)$$

In this formulation, the gauge condition is handled by specifying the magnetic field on the boundary using the Biot-Savart law. This can be rewritten as a boundary condition on Ψ and is given by:

$$\nabla \Psi \cdot \mathbf{n} = \left[\mathbf{T} - \left(\frac{1}{4\pi} \left(\sum_{k=1}^{\text{elements}} I_k e^{j\phi_k} \int \frac{d\mathbf{l}_k \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} + \sum_{i=1}^{\text{elements}} \int_{\Omega_i^c} \frac{(\nabla \times \mathbf{T}) \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dV \right) \right) \right] \cdot \mathbf{n} \quad (8)$$

where I_k and ϕ_k is the the amplitude and phase shift of the multiphase current in the k^{th} coil turn, $d\mathbf{l}_k$ is an element of length along the coil turn, dV is a volume element in the conducting domain, and $|\mathbf{r} - \mathbf{r}'|$ is the distance from a point in the metal to the respective integration element. The advantage of this integral formulation is that it allows the incorporation of complicated coil configuration. In other words, the location and current of each induction coil may be individually specified.

At steady state, the differential equation representing the heat transfer in the conducting domain is given by the Fourier's law:

$$\rho C_p (U \cdot \nabla T) = \nabla \cdot (k \nabla T) + \frac{J^2}{2.0 * \sigma} \quad (9)$$

where U is the speed, and C_p is the temperature dependent specific heat of the metal, and J is the magnitude of the current density.

3. Numerical Results

The formulation described above constitutes the computational core of a finite element methodology for coupled three-dimensional electromagnetic and thermal problem. In this section, the predicted values of magnetic and current field is compared against analytical solution for a rectangular body in a uniform time varying magnetic field over a range of magnetic interaction parameter. After the validation of the electromagnetic field calculation, the numerical methodology is used to predict the current field, Joule heating, and the thermal field in the practical case of a transverse flux induction heating furnace.

3.1 Applied Time-Varying Magnetic Field

Figure 1 shows an infinitely long sheet with rectangular cross sectional area in a uniform time-varying magnetic field B_0 applied in the positive y-direction. This constitutes a three-dimensional, time harmonic eddy current problem which is similar to the transverse flux induction heating. In this test case, the origin of the coordinate system is placed at the geometric center of the sheet, which has a conductivity of 5×10^6 mhos and magnetic permeability of $4\pi \times 10^{-7}$ (H/m). The magnetic interaction parameter is varied from 1 to 4 by varying the frequency of the applied field from 6 kHz to 30 kHz.

Since a three-dimensional analytical solution to this problem is difficult to obtain, a one-dimensional solution of this problem is obtained by making the assumption that the sheet

is infinite in the x and z direction. Therefore, the solution for the magnetic flux density and induced current density depends only on the y -coordinate. For such a system, analytical values of magnetic flux density and the induced current density distribution in the metal sheet is available in literature [6].

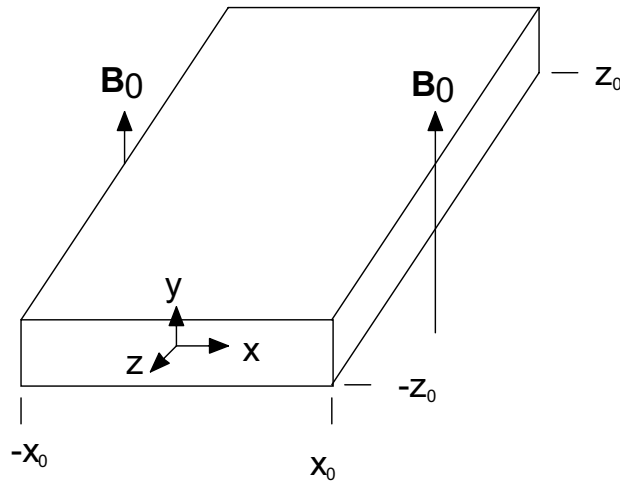


Figure 1. Schematic sketch of the applied magnetic field system

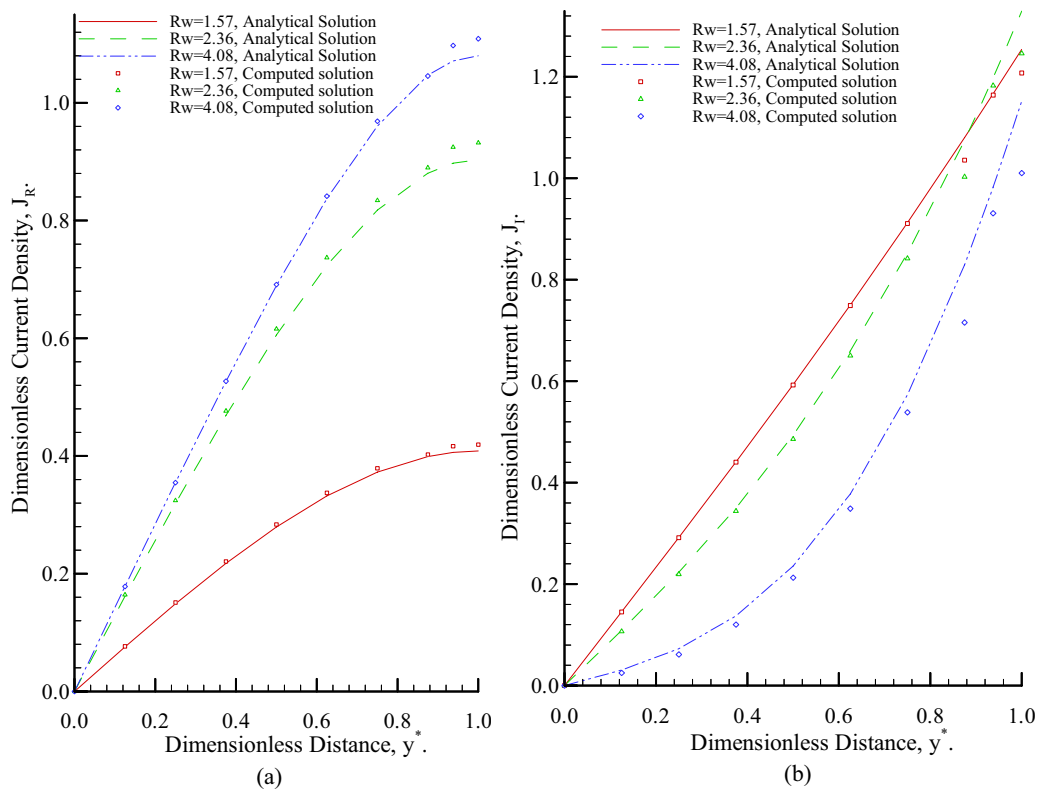


Figure 2. Comparison of theoretically predicted and numerically computed variation of induced current density for a range of magnetic interaction parameter: (a) real part, and (b) imaginary part.

Figures (2) and (3) show the results of the finite element computation where the comparison of the quantities is made at $x=z=0$. This way the influence of the ends is at a minimum and allows us to compare the three-dimensional computed solution with the one-

dimensional analytical solution. The magnetic field was normalized with respect to the applied magnetic field, B_0 , while the induced current density is normalized with respect to J_{ref} ($J_{ref} = \frac{B_0}{\mu\delta}$) which is derived from the differential form of the Ampere's law.

Figure (2) compares the analytical and computed current density at three different frequencies for a thin metal sheet of aspect ratio of 10. The maximum error was of the order of 3%. The accuracy can be improved by increasing the number of nodes and the aspect ratio. The error is also seen to increase with frequency but this can be alleviated by increasing the number of grid points in the skin depth. Figure (3) shows that the computed magnetic field at the surface slightly deviates from the analytical solution and can be attributed to the methodology of calculating the magnetic field using the Biot-Savart law.

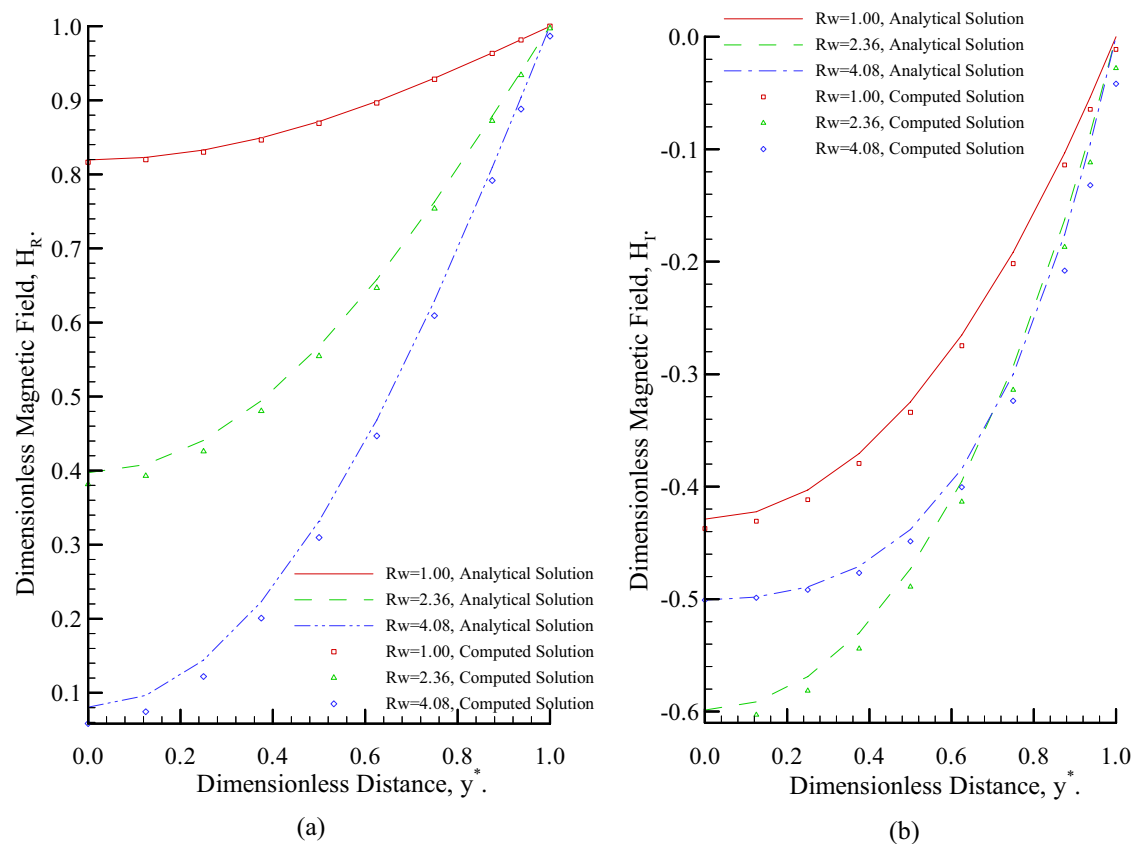


Figure 3. Comparison of theoretically predicted and numerically computed variation of the total magnetic field at different magnetic interaction parameter: (a) real part, and (b) imaginary part.

3.2 Transverse Flux Induction Heating

In order to examine the ability of the solution technique to solve both the electromagnetic and thermal problem, the case of transverse flux induction heating was examined. Figure (4) shows a three-dimensional schematic sketch of a portion of the transverse flux induction coil. This figure shows the expected direction of the magnetic and current field and direction of the steel strip movement. In the steel industry, a cold rolled steel strip about 1 mm in thickness and 1 m wide is run at about 2 m/s through an induction galvanneal furnace about 1m in length. For these conditions, calculations were run at two

different coil frequencies of 8 KHz and 16 KHz and for each frequency two different coil currents of 200 A and 342 A were considered. The results for 200 A and 8 KHz is presented.

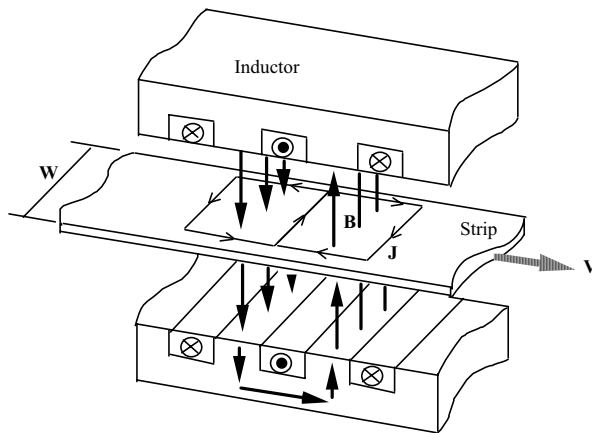


Figure 4. Three-dimensional sketch of a typical transverse flux induction heating furnace.

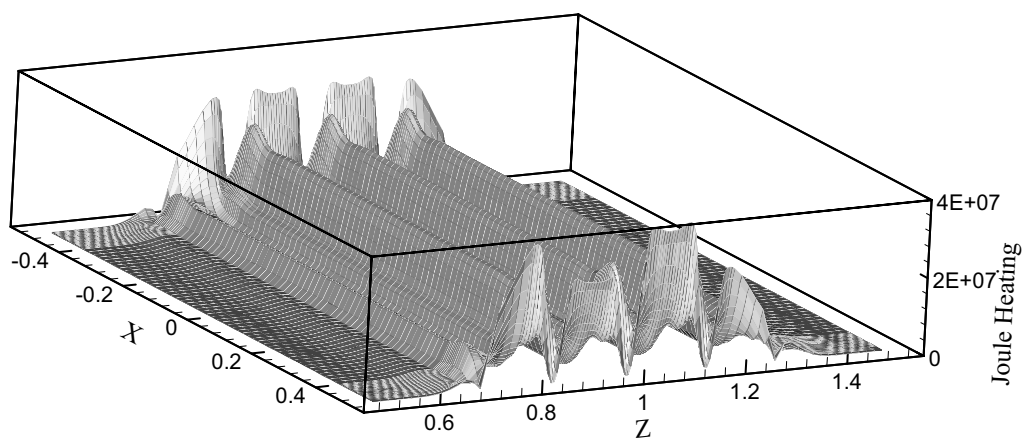


Figure 5. Predicted values of Joule heating along the length and width of the strip.

The current density is highest at the edges of the strip and under the slot and is expected to lead to an increased Joule heating effect, as shown in Figure 5. Using the calculated value of Joule heating, the temperature along the length and width of the strip was calculated. The temperature in the thickness direction of the strip should be uniform because of the thermal conductivity and thickness of steel strip, and minimal decay of the current density in the thickness direction. Figure 6 shows the temperature profile along the width in the induction furnace at steady state. It can be seen that the temperature increases by 10 C along the length of the furnace and does not exceed 515 C which is the upper limit of good galvaneal. It can also be seen that the temperature of the strip along the width is uniform except at the edges of the strip.

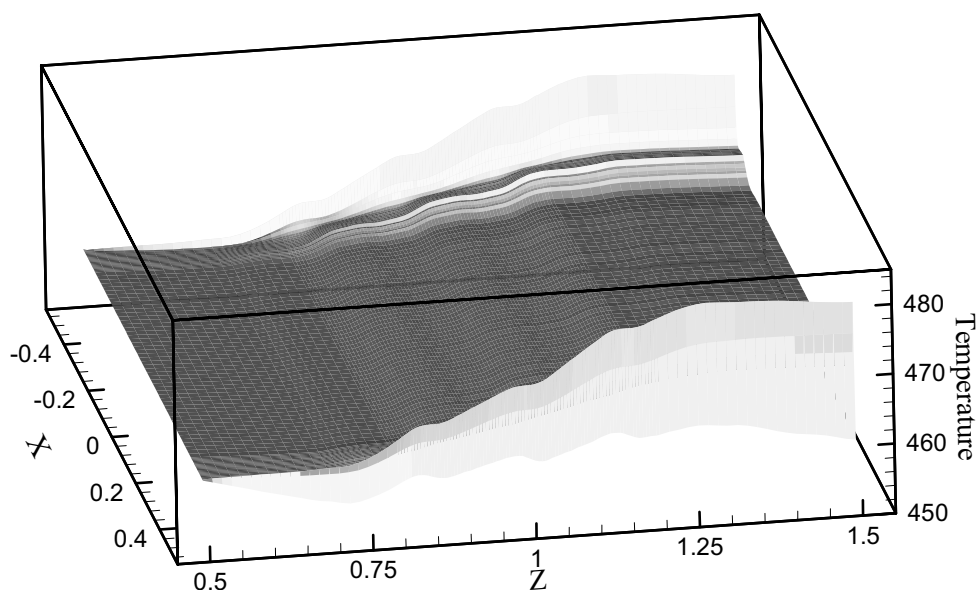


Figure 6. Temperature Surface Plot

4. Conclusion

A numerical methodology was developed to solve coupled electromagnetic and thermal problem which restricted the solution domain to the conducting region. The methodology was tested against one-dimensional problem of a rectangular conductor subjected to a transverse magnetic field. After verifying the accuracy of the methodology, the case of a steel strip heated by a transverse flux induction heating coil was investigated.

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