International Scientific Colloquium
Modelling for Electromagnetic Processing
Hannover, October 27-29, 2008

Electromagnetic Stirring Effect on Thermal Conductivity of a Levitated Sample

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Abstract

Modulation calorimetry is an indirect method to measure specific heat capacity and thermal conductivity. When using electromagnetic levitation, this technique allows the measurement for highly reactive molten and undercooled alloys. This work presents 2D-axisymetric simulations showing evidences of convective effects on the sample temperatures. Evidence of heat transfer in the sample is observed, leading to an overestimation of the thermal conductivity. No negative impact has been observed on specific heat capacity measurement.

Introduction

The modulated calorimetry using inductive levitation consists in deducing thermophysical properties of a sample placed in an alternating magnetic field from its thermal behavior. Induced currents generate a total Joule power *P* which amplitude is modulated. Polar surface temperature is recorded. This signal is then analyzed using an analytical model to calculate specific heat capacity and thermal conductivity.

In this article, we present the experimental device and briefly sum up the modulation calorimetry technique. Thermal behavior of levitated sample is deduced from unsteady thermal simulation [1] Electromagnetic stirring effect on heat transfer is deduced from the comparison between solid and liquid results.

1. Modulation calorimetry measurement

1.1 Principle

A spherical sample of radius R and electrical conductivity σ_{el} is located in an inductor powered by an alternating current ($\omega_1/2\pi = 350$ kHz). This inductor generates a magnetic field which induces, inside the sample, electrical currents localized in a surface layer called the electromagnetic skin depth δ such that :

$$\delta = \sqrt{\frac{2}{\mu_o \omega_1 \sigma_{el}}} \tag{1}$$

where $\mu_o = 4\pi 10^{-7} H.m^{-1}$ is the vacuum permeability. Those currents compose with the magnetic field creating a force field able to center the sample. They also generate a Joule heating power P inside the sample. If the sample is liquid, the body force causes an electromagnetic stirring. Resulting velocity in the sample is of the order of magnitude of the Alfvèn velocity $U_A = I_0 C\sqrt{\mu_0/\rho}$, where I0 is the peak current in the inductor, r is the density and C = 170.1 is a geometrical factor related to the inductor geometry described in figure 3. Reynolds number in such a system is defined as $R_A = U_A R/\nu$ where is the cinematic viscosity.

Modulated calorimetry consists in perturbing the thermal equilibrium of the sample by modulating the total Joule power P around its mean value $P = \overline{P} + \widetilde{P}$, where $\widetilde{P}$ is the fluctuating part of the Joule power assumed to be such that $\widetilde{P} = \alpha\overline{P}\sin(\omega_2 t)$, α being the relative perturbation amplitude and ω_2 the modulation angular frequency, negligible (0.01 to 100 Hz) compared to ω_1, the angular frequency of the inducting current. For clarity sake, all time dependant variable $X(t)$, the following convention is taken:

$$X(t) = \overline{X} + \widetilde{X} \tag{2.a}$$

where $\overline{X}$, $\widetilde{X}$ are the time average and the fluctuating parts respectively.

$$\overline{X} = \frac{1}{t_X}\int_0^{t_X} X(t).dt \quad \text{and} \quad \widetilde{X} = X(t) - \overline{X} \tag{2.b}$$

where t_X is the duration of the experiment. For an established harmonic regime, the resulting polar temperatures $T_p(t)$ can be written in the same way :

$$T_p(t) = \overline{T}_p + T_{p,o}\sin(\omega_2 t + \varphi_p) \tag{2}$$

Figure 1 is a schematic representation of the principle of the measure. In experiments, $T_p(t)$ is measured at the sample surface by using a pyrometer. The sinusoidal modulation of the power, of $\alpha\overline{P}$ amplitude, generates an oscillation of the polar temperature of $T_{p,o}$ amplitude around its mean value $\overline{T}_p$. The principle of the analysis consists in measuring the ratio $T_{p,o}/\alpha\overline{P}$ for a set of angular frequency ω_2.

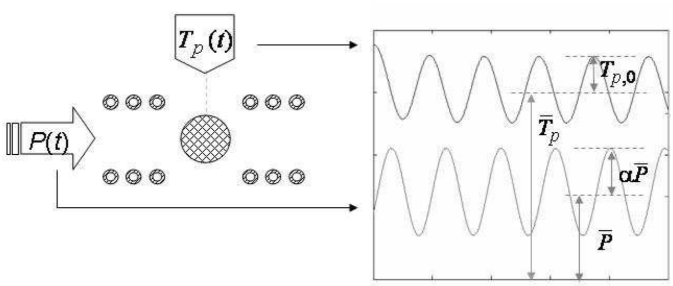

Fig. 1: schematic representation of the sample inside the inductor and measurement et principle

1.2 Heat transfer model

Modulation calorimetry is an indirect technique to determine thermophysical properties, and as such, quality of the measurement is bound to the model used to describe the thermal behavior of the sample. The analytical model used to find C_p and κ_{th} is related to a simplified description of the sample thermal transfers as proposed by Fecht in [2]. It describes the sample by two geometrical domains. An equatorial domain, of g_e volume ratio and s_e external surface ratio, is receiving the whole Joule power while a polar domain, of (1-g_e) and (1-s_e) volume and surface ratio, is heated by conduction with the equatorial domain. Those domains are assumed to be isothermal. External heat transfer are assumed radiative only and modeled by a global radiative heat transfer $h_{ext} = 4A\varepsilon\sigma\overline{T}^3$ where A is the sample surface, ε is the total hemispherical emissivity, σ the Stefan-Boltzmann constant and $\overline{T}$ the surface mean temperature of the sample. Heat transfer between the two domains are modeled by a global conductive heat transfer $h_{int} = 4\pi(R-\gamma\delta)\kappa_{th}$ where γ is a geometrical factor. The total Biot number of the system Bi is then defined as follows:

$$Bi = h_{ext}/h_{int} \tag{4}$$

It lead to the equation system (7) :

$$\begin{cases} C_p g_e \dot{\widetilde{T}}_e = h_{int}\left[\widetilde{T}_p - (1+s_e Bi)\widetilde{T}_e\right] + \widetilde{P} \\ C_p(1-g_e)\dot{\widetilde{T}}_p = h_{int}\left[\widetilde{T}_e - (1+(1-s_e)Bi)\widetilde{T}_p\right] \end{cases} \tag{5}$$

Heating power fluctuation is considered harmonic of angular frequency ω_2 :

$$\widetilde{P} = \alpha\overline{P}\cos(\omega_2 t) \tag{6}$$

Analytical solution of equation (5) and equation (6) for the polar temperature fluctuation leads to:

$$\frac{\widetilde{T}_{p,o}}{\alpha\overline{P}} = \frac{h_c}{C_p^2 g_e(1-g_e)} \frac{1}{\left[(\lambda_{int}{}^2 + \omega_2^2)(\lambda_{ext}{}^2 + \omega_2^2)\right]^{1/2}} \tag{7}$$

Where λ_{ext} and λ_{int} are typical frequencies related to external and internal heat transfer respectively. For small Biot number, their expression can be approximated by

$$\lambda_{ext} \cong h_{ext}/C_p \tag{3}$$

$$\lambda_{int} \cong g_e(1-g_e)h_{int}/C_p \tag{9}$$

Indirect measurement of the properties consists in identifying the above described heat transfer model to the experimental measurement. The present study focus on the impact of the electromagnetically driven steady fluid flow measured thermal behavior itself.

2 Results

2.1 Numerical simulation conditions

Some general assumptions are made in order to separate the different simulations:

- Electrical, mechanical and thermal properties are assumed to be constant in the range of temperature of the sample.
- Alfvèn velocity is such that the magnetic Reynolds $R_m = \mu_0 \sigma_{el} U_A << 1$. Consequently, velocity can be neglected in the induction equation.

The following points make possible to simulate a separation between the different systems. Assumptions are summed up in figure (2). Consequently, the different simulations are performed consecutively

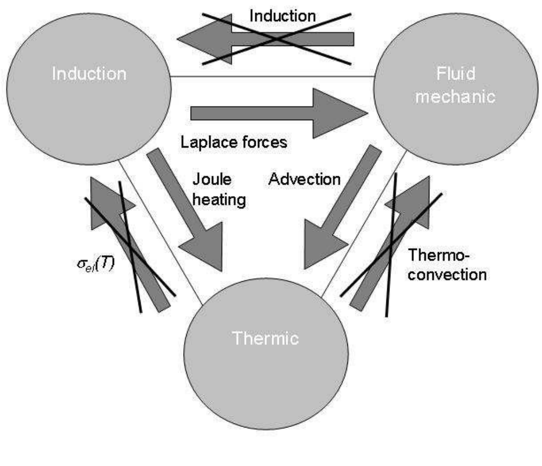

Fig. 2 : Physical coupling scheme

Induction equations are solved by using a harmonic formalism of the electromagnetic field in axisymmetric 2D thanks to a module developed within EPM laboratory [3] The sample is a solid sphere of constant electrical conductivity σ_{el}. Inductor geometry is described in figure 3. Results of this simulation are $\overline{\chi}$ and $\bar{\boldsymbol{f}}$, the Joule heating density and the Laplace body force respectively. Steady fluid flow is solved for a spherical drop stirred by the body force $\bar{\boldsymbol{f}}$. Reynolds number Re_A is always low enough for the fluid flow to be laminar. Steady velocity field $\overline{\boldsymbol{U}}$ is obtained.

Unsteady temperature field is solved for both solid and liquid cases. Equations are heat transfer equations in 2D axisymetric. Velocity field $\boldsymbol{U}$ is assumed to be constant and equal to $\overline{\boldsymbol{U}}$ despite the modulation. Boundary heat flux corresponds to a grey and diffuse surface in a black body enclosure at room temperature $T_o = 300K$.

$$\begin{cases} \dfrac{\partial T}{\partial t} - \dfrac{\kappa_{th}}{\rho c_p}\Delta T + \dfrac{1}{\rho c_p} U\nabla T = \dfrac{\chi}{\rho c_p} & (r,\theta) \in [0;R]\times[0;\pi] \\ \kappa_{th}\dfrac{\partial T}{\partial r} = \varepsilon\sigma(T^4 - T_0^4) & (r,\theta) \in R\times[0;\pi] \end{cases} \tag{4}$$

Heat source is $\chi = \overline{\chi}(1+\alpha\, g(t))$, where the relative modulation amplitude α is 5 % and *g(t)* the modulation function. Modulation function is fully described in [4]. The resulting time dependant polar temperature is analyzed and its thermal behavior is obtained. Simulation parameters are summed up in table 1.

Tab. 1. Simulations parameters

Sample radius *R, mm*	4
density ρ, $kg.m^{-3}$	3860
heat capacity c_p, $J.K^{-1}.kg^{-1}$	663
thermal conductivity κ_{th}, $W.m^{-1}.K^{-1}$	23.1
electrical conductivity σ_{el}, $\Omega^{1}.m^{-1}$	$5.26\ 10^5$
total hemispherical emissivity ε	0.4
inductor current I_0, *A.turn*	390
currents angular frequency ω_1, $rad.s^{-1}$	$2\pi\ 350.10^3$
Alfvèn velocity $m.\ s^{-1}$	$9.3\ 10^{-6}$
Biot (solid)	0,06
Reynolds	37

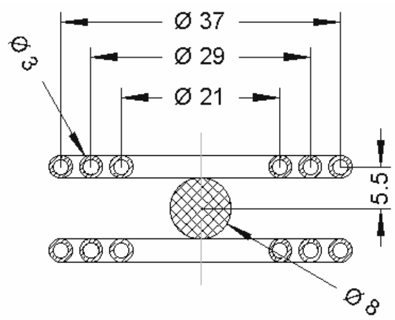

Fig. 3 : Sample and inductor geometries

Resulting ratio $T_{p,o} / \alpha\overline{P}$ is reported in figure (4). Solid line represent the thermal behavior of the solid sample, dashed line is the liquid sample. Slopes are reported on both curves. The two domains model is known to be a very good representation of a solid sample behavior [1]. As such, the simulated solid sample thermal behavior demonstrates a trend similar to the analytical solution given in equation (7). Three ranges of angular frequencies are observed, each one separated by natural frequencies λ_{ext} and $\lambda_{int.}$ In the case of the liquid sample, we clearly see a shift toward the higher frequencies, demonstrating the impact of the convection on internal heat transfer. Measured internal frequencies are 9.0 and 25.7 rad.s^{-1} for solid and liquid respectively.

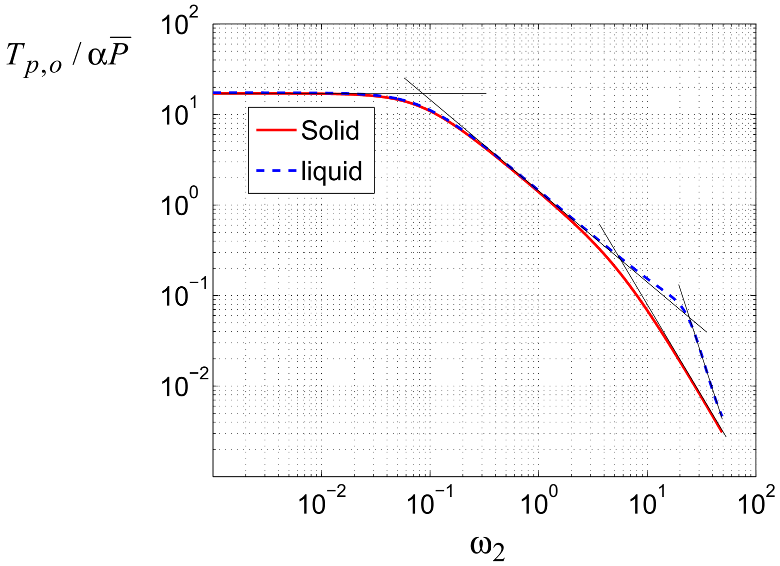

Fig. 4 : Simulated thermal behavior

This results demonstrates the negative impact of convection on intrinsic thermal conductivity in the modulation calorimetry method. Efforts should be made to develop accurate inverse heat transfer model, either analytical or numerical, taking into account electromagneticly driven convection.

Acknowledgement

This work has been funded by the followings supports : ESA under MAP-Thermolab, EC under PI-IMPRESS (Contract Number: NMP3-CT-2004-500635), CNES under Material Sciences Program.

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