International Scientific Colloquium
Modelling for Electromagnetic Processing
Hannover, October 27-29, 2008

Thermoconvective instabilities of molten glass heated by direct induction in a cold crucible

E. Sauvage[1, 2 *], A. Gagnoud[1], Y. Fautrelle[1], P. Brun[2], J. Lacombe[2]

Abstract

The aim of this study is to perform numerical simulation of a vitrification process developed by the French atomic commission. This process uses direct induction technology in a cold crucible to melt specific glass at high temperature (~1250°C).

The numerical simulation of this process needs a coupled approach of the different phenomena: induction, thermal and hydrodynamic. Indeed, those three phenomena are strongly coupled because of the temperature dependence of the glass properties. For example, the hotter the molten glass, the higher the electrical conductivity.

Two softwares are used to achieve the purpose of the study. Thermal and hydrodynamic aspects are modeled with Fluent® whereas induction is modeled with Flux®. A home-made iterative coupling is performed between the two softwares which leads to a full 3D simulation of the molten glass heated by induction.

Buoyancy and Marangoni convection are taken into account. Thermoconvective instabilities appear within the glass bath when the total Joule power injected reaches a specific threshold. Qualitative comparison of the aspect of free surface is performed. The direct measurement of the temperature in the glass bath gives a quantitative comparison between experimental and numerical results. Reasonably good agreement is found in both cases.

Introduction

A direct induction process using the cold crucible technology has been developed in the French Atomic Commissariat for 35 years. In this process, walls of the crucible are cooled by internal water circulation which protect them from the corrosion and allow higher temperature of the molten glass. Moreover, a mechanical stirrer and air bubbling system promote a good homogeneity of the melt. In results, the lifetime of the process as the incorporation rate of the nuclear waste is increased.

This study is a continuation of previous works modeling this process (Jacoutot [1,2]). The aim is to model the molten glass heated by direct induction. Thus, hydrodynamic, thermal and induction phenomena are taken into account. As the physical properties of the glass are functions of the temperature, these three phenomena are strongly coupled. Consequently, a coupling between two softwares is achieved. Fluent® is used to solve hydrodynamic and thermal equations whereas Flux® computes the Maxwell equations. All computations are three-dimensional. The coupling is based on files data transfers. Each software interpolates the field on the computing nodes of the other software.

Results are compared with those of Jacoutot [2] obtained with simulation in which induction heating is solved under axisymmetric assumption. This former coupling will be called 'pseudo-3D coupling'.

* corresponding author : emilien.sauvage@cea.fr, +(33) 4 76 82 52 55, +(33) 4 76 82 52 49

1. Modelisation

1.1. Geometry

The molten glass bath is a cylinder of 185mm high and 250mm radius. In this study the mechanical stirrer is not modeled, only four thermocouples within a sheath made of alumina (electric insulated) are taken into account.

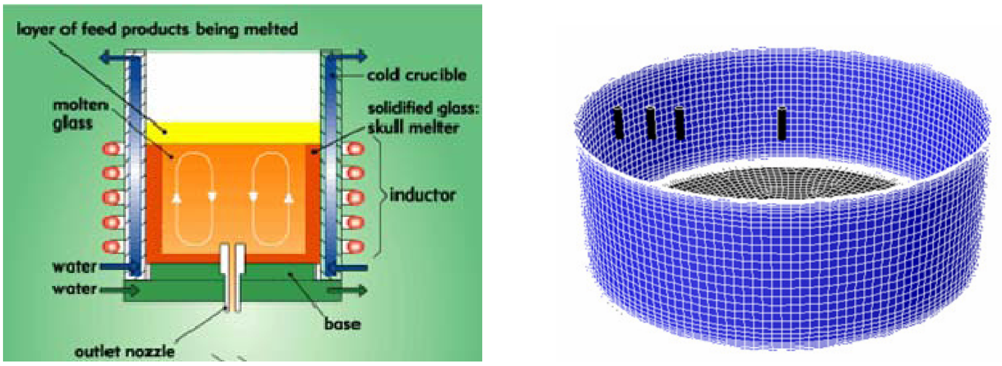


Fig. 1: *Process overview (left) and Fluent® mesh (right). The four black elements are sheath for thermocouples made of alumina.*

1.2. Electromagnetic model

The commercial software Flux® is used to solve induction equations. The crucible as well the inductor are not modeled, they are approximated by a current sheet around the glass. In this coil, an alternative current (I_{eff} = 1500 A and 280 kHz) is imposed. Owing to the high value of the frequency, the quasi-steady approximation is made. Different formulations for induction equations are available. The A-V formulation gives best result in a material with high gradient of electrical conductivity.

$$\vec{\nabla}\times\left(\frac{1}{\mu_0}\vec{\nabla}\times\left(\vec{A}\right)\right)+\sigma\left(\frac{\partial\vec{A}}{\partial t}+\vec{\nabla}V\right)=\vec{0}$$
$$\vec{\nabla}\cdot\left(\sigma\left(\frac{\partial\vec{A}}{\partial t}+\vec{\nabla}V\right)\right)=\vec{0} \qquad (1)$$

The coil is not meshed so a reduced scalar potential formulation is used. The resolution is achieved with iterative methods such as a conjugate gradient one. The domain is discretised with approximately 200 000 first-order elements.

1.3. Thermal-Hydrodynamic model

Fluent® software solves the Navier-Stokes and thermal equations. The flow is assumed to be laminar due to the high viscosity of the glass.

$$\vec{\nabla}\cdot\vec{u}=0$$
$$\rho_0\left(\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}\right)=-\vec{\nabla}p+\vec{\nabla}\overline{\overline{\tau}}-\rho_0\beta\left(T-T_0\right)\vec{g} \qquad (2)$$

$$\overline{\overline{\tau}}=\vec{\nabla}\cdot\left(\mu\left(\vec{\nabla}\cdot\vec{u}+\vec{\nabla}\cdot\vec{u}^T\right)\right) \qquad (3)$$

$$\rho_0 \left(\frac{\partial c_P T}{\partial t} + \left(\vec{u} \cdot \vec{\nabla} \right) c_P T \right) = -\vec{\nabla} \cdot \left(\lambda \vec{\nabla} T \right) + Q_{th} \quad (4)$$

The source terms in the right hand side of equation (4) is the Joule power density dissipated in the glass. This term is calculated by Flux® as a function of induced current:

$$Q_{th} = j/_{2\sigma} \quad and \quad \vec{j} = \vec{\nabla} \wedge \vec{T} \quad (5)$$

The convection is driven by two phenomena. First, buoyancy forces are modeled in the Boussinesq approximation (last term of right hand side of equation (2)). Second, thermocapillary convection at the surface is taken into account. This effect is due to the variation of the surface tension with the temperature. This dependence is well described with the law: $\sigma_s(T) = \sigma_{s0} - \gamma(T - T_0)$ where $\gamma = -\partial\sigma_s/\partial T$. For almost all liquid, γ is constant and positive and for the glass its value is $10^{-4}\ N.m^{-1}.K^{-1}$ [3]. The boundary condition at the free surface relates the viscous strain with the thermal strain in the direction of the surface, for example, in the radial direction:

$$\mu \left. \frac{\partial u}{\partial z} \right|_{surf} = \gamma \left. \frac{\partial T}{\partial r} \right|_{surf} \quad (6)$$

Thermal boundary conditions are handled via a global exchange coefficient on the crucible wall and bottom. At the free surface a mixed condition convection-radiation is considered with an emissivity of 0.9. The glass is supposed to be a Newtonian fluid but all the physical properties are complex functions of the temperature. These laws are confidential but Table 1 summarizes order of magnitude for two temperatures. The confinement glass of this study is opaque so there is no need of internal radiation model. In fact, the internal radiation is naturally included in the variation of thermal conductivity with temperature.

Physical properties	**Unity**	**500 K**	**1500 K**
Electrical conductivity	$\Omega^{1}.m^{-1}$	10^{-4}	20
Dynamic viscosity	$Pa.s$	10^{14}	1
Specific heat	$J.kg^{-1}.K^{-1}$	900	1500
Thermal conductivity	$W.m^{-1}.K^{-1}$	1	6
Density	$kg.m^{-3}$	2850	2750

Tab. 1: *Order of magnitude of physical properties of the glass*

2. Coupling Strategy

2.1. Mesh to mesh interpolation

The main issue is to couple a finite element and a volume finite based software. Mesh refinement requirements are different for the induction and hydrodynamic phenomena. Consequently, using a unique mesh is not possible and interpolations between the two meshes are done to minimize losses of precision. These methods have to be robust and fast in order to not penalize computation time. Flux® software natively includes these methods but specific functions have been created to allow Fluent® to do these interpolations using the calculated temperature gradient in each cell.

2.2. Coupling strategy

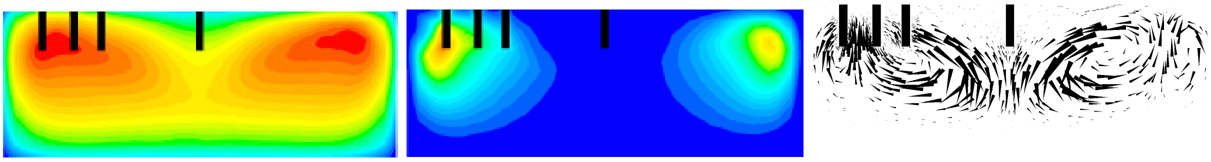
The Joule power distribution is calculated every 20s second of flow computation. This time was chosen in regard of the diffusion time which is 10^3 *s*. As Jacoutot shown, this does not change the results. The time step for Fluent® is 1s.
Initialization is performed with the results obtained with the pseudo-3D coupling due to Jacoutot [2]. In this coupling, the electromagnetic computation was axisymmetric.

The convergence criterion for the iterative coupling is based on the square error between the old and new map of temperature. It is calculated by the software Flux® during importation of the new field of temperature, the convergence threshold is set at 10^{-8}.

3. Results

3.1. Total Joule power injected of 45kW

In this first case, a 45kW Joule power heating is imposed in the glass container. Figure 2 shows iso-contours of temperature and Joule heating in a vertical plane including thermocouples. Cold glass in contact with the cold crucible appears clearly. The maximum Joule power density is located in the hot parts of the bath which have a better electrical conductivity. For such a Joule heating power, glass flow due to natural convection and thermocapillarity is stable and stationary. Without thermocouples, an axisymmetric flow appears. Such a result has already been reported by Jacoutot [1]. 3D simulations only exhibit a small perturbation of the Joule heating field near the thermocouples.



(a) (b) (c)

Fig. 2: *Temperature isocontours (a), Joule power density isocontours (b) and velocity vectors (c) on crossing vertical plane including thermocouples. Scales are respectively [300;1670]K, [0 ;7,76.10^6]W.m^{-3} and [0 ;4,68.10^{-4}] m.s^{-1}*

Figure 3 shows how induced current in the molten glass avoid the sheath of thermocouple made of alumina (electrical insulated). The Joule power then concentrates on both side of the sheath creating two local overheating zones.

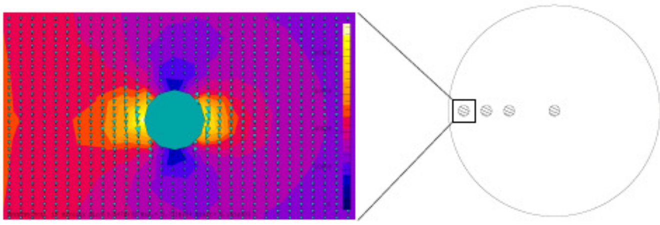


Fig. 3: *Joule power isocontours and induced current vectors at the free surface near thermocouple TC3.*

3.2. Total Joule power injected of 55kW

For this total Joule power rate, the flow obtained with the pseudo-3D coupling exhibits thermoconvectives instabilities [2]. These instabilities are convection cells of rising hot glass similar to Bénard-Marangoni cells [4] (*cf.* figure 4).

When the 3D-3D coupling is performed, the Joule power concentrates in the hotter glass zone increasing temperature differences. Thus, the instability is reinforced, and the convections cell growths and their number decreased. Moreover, this instability was initially stationary (non time dependent). The figure 5 shows the evolution of the number of convective cells with physical time. The initialization exhibits 22 cells. During the first 1000s, this number decreased to 11 because cells growths, then cells number decrease a second time because of loss of periodicity, and cells appears and disappears in the glass. Cell number oscillates between 5 and 9.

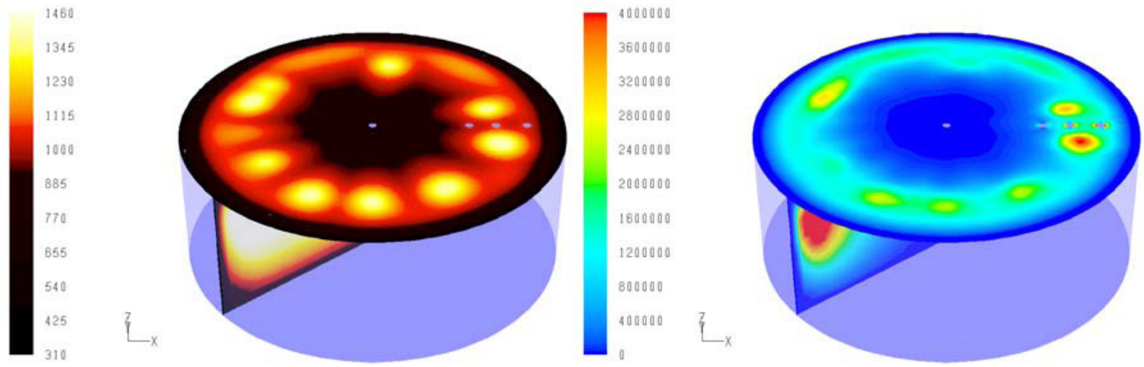


Fig. 4: *Temperature isocontours (left) and Joule power density isocontours (right) at the free surface and in a crossing vertical plane. Scales are respectively [310;1450]K and* $[0\ ;4.10^{6}]W.m^{-3}$.

Unfortunately, the construction of the classical non-dimensional numbers as Rayleigh or Marangoni is very difficult because physical properties of the glass are strongly dependent of the temperature.

Characteristic time of evolution of convective cell is about ~15min which is in qualitative agreement with experimental visualization. Number of instabilities in the glass could not be determined experimentally because of the limited view angle of the camera, but size of cells (~10cm) is in agreement (*cf.* figure 6).

Figure 7 shows temperature measurement with the thermocouples compared with calculated temperature with the pseudo-3D and 3D-3D coupling. Results are in good agreement too.

Conclusion

The 3D coupling enlightens the flow instability, that may occur in the molten glass heated by induction. It has been demonstrated that three-dimensional effect of induction heating could have a significant impact on the flow configuration. If a certain threshold of the total injected power is reached, then 3D thermoconvective cells appear. This instability becomes unsteady and presents chaotic behavior. The wave numbers and also characteristic times are in good agreement with the experimental data, obtained in CEA Marcoule. However, due to the difficulty inherent to the choice of reference values in term of thermophysical properties, it was not been possible, yet, to derive relevant non-dimensional number.

References

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Acknowledgment

We wish to thank the Cedrat company for the loan of their software Flux®.
Web: www.cedrat.com Email: cedrat@cedrat.com

Authors

(1) SIMAP - EPM laboratory
ENSEEG BP 75
38402 Saint Martin d'Hères
France

(2) CEA – Marcoule
SCDV/LDPV BP 17171
30207 Bagnols sur Cèze Cedex
France

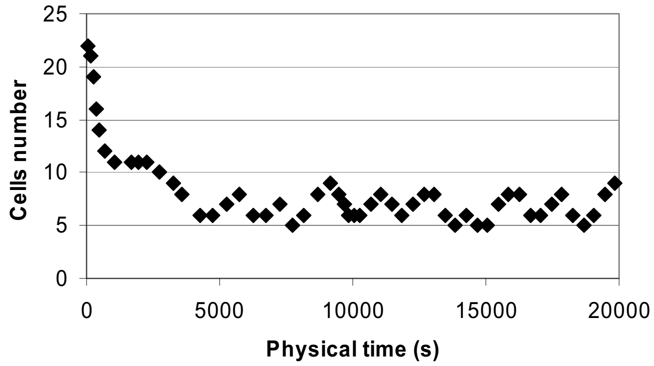

Fig. 5: *Time evolution of the number of convection cells.*

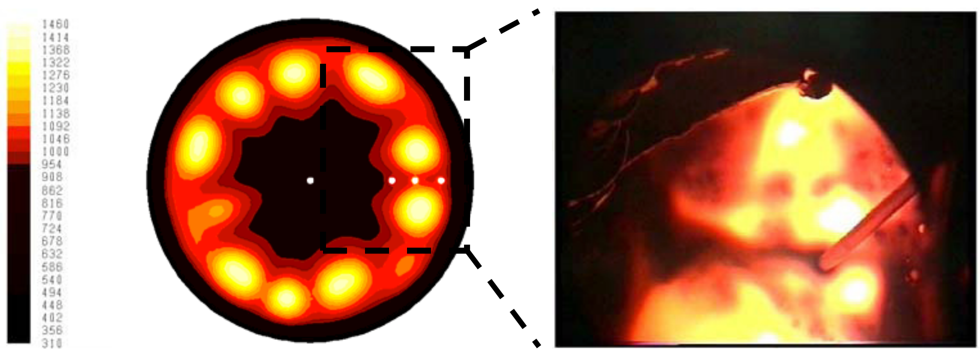

Fig. 6: *Numerical (left) and experimental (right) temperature isocontours at the free surface of the molten glass.*

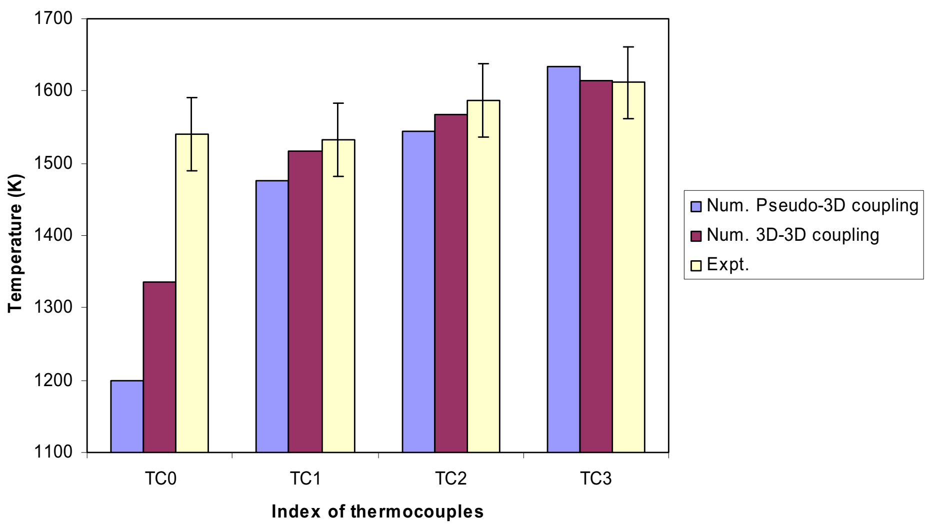

Fig. 7: *Comparison of measured temperature and calculated ones at the location of the thermocouples.*