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Thermo-Mechanical Fatigue Life Estimation of Induction Coils

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Abstract

As industrial induction heating systems get increasingly automated and the power supplies get more controllable with decreased ramp-up and -down times, heat cycles can be shortened. This potentially leads to increase in heat cycles per time and in some cases, shorter inductor life. Some of these failures are expected to be caused by thermal fatigue. A model for coil service life estimation with regards to thermal fatigue is suggested. Three modeling stages are suggested, temperature-, mechanical- , and fatigue stage. An example is calculated using the models and an estimate of the service life is found.

Introduction

In induction heating applications the induction coil is a component of crucial importance. The life expectancies of these units are depending on the environment the coils are exposed to and the application where they are used. A properly designed coil in a less demanding application may have a very long service life. Proper maintenance as described in [1] is important in many applications and environments. There are many identified modes of failure [2, 3]. Still, in some applications, there are coils that have much shorter service life than expected, even if they seem to have been properly designed and maintained. Typically coils used in applications that imply high current densities and short heat cycles have shown to fail prematurely. In these cases thermal fatigue is expected to be a limiting factor of service life. To our knowledge publications on thermal fatigue of induction coils have been very limited. To investigate this thermal fatigue properly, a good understanding of Joule loss, heat transfer and temperature distribution in the coil is a natural first step. A model combining the magneto thermal aspects of the losses in the coil with the thermal fluid dynamic aspects of the water cooling was presented [4]. The model provides thermal information of the cross section of the coil. In this paper the information from the thermal model of [4] is used in a mechanical model to provide internal stress/strain in the coil. Finally lifetime estimations can be found from fatigue curves for the material used in the coil.

1. Temperature Modeling

An induction coil is inherently a three dimensional structure and the models of real life coils would also need to take three dimensional effects into account. For instance the coolant temperature increases along the coil, coolant velocity profile develops after a change in direction, size, or shape of the coil cooling channel, mechanical stress causes movement and torsional or bending stress in other regions of the coil. This leads to complex models that would be difficult to make and difficult to verify, so simplified models are needed. To simplify the models, a near two dimensional plane symmetric geometry is investigated. Radiation and cooling by the surrounding air are neglected. Two straight parallel tubes with the same cross section are placed 2 mm apart as shown in Fig. 1. The same electric current

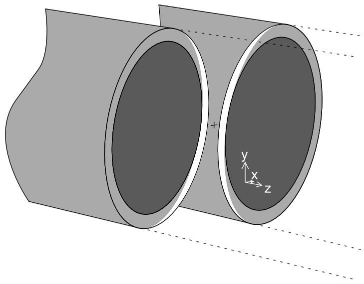

Fig. 1. Cross section of conductors with heating zones in white. The cross in the middle between the centres of the tubes is origin of the coordinate system

passes through the two coil conductors in opposite directions. The stationary temperature profile of the coil conductor is a result of the balance between the Joule losses generated by the electric current and the heat removed by the coolant. Both the Joule losses and the fluid flow are depending on the temperature of the coil conductor, so a solution has to be refined through an iterative process until a solution that satisfies both domains is found. The model used consists of separate models for the magneto thermal and for the fluid dynamic aspects of the coil and external scripts for the iteration process as described in [4]. The magneto thermal domain is modeled in the finite element based Flux 2D and finite volume based Fluent 3D is utilized for the fluid dynamic aspects. This model produces an estimate of the temperature across the cross section as seen in Fig. 2.

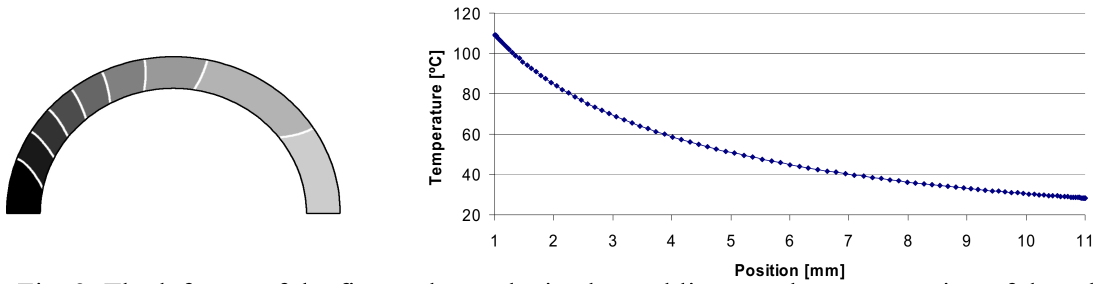

Fig. 2. The left part of the figure shows the isothermal lines on the cross section of the tube (30 to 100 °C in steps of 10 °C from right to left). The graph shows the temperature on the upper surface of the tube as a function of position x for the simulation.

An experimental setup was designed and constructed to validate the temperature profiles found with the model, by comparison to the surface temperature as obtained by a thermal imaging camera. The setup was constructed in an induction hardening machine. A hairpin coil, held in place by a glass fiber reinforced polyester frame was used to prevent the magnetic and thermal expansion forces from pulling the coil out of shape.

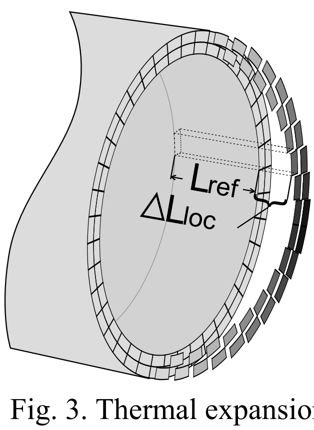

Fig. 3. Thermal expansion of free moving finite elements.

2. Stress and Strain Modeling

2.1. Thermal Expansion

The previous model provides temperature information of the cross section of the coil. In the following a focus is put on the resulting stresses and strains caused by the change in temperature. The thermal model is based on no motion in the x-direction (2 mm constant gap between the two conductors or legs of the coil) and the motion in the y-direction is negligible, but there is no limitation on motion in the direction of the current flow. This reduces the stress/strain model also to a two dimensional model with motion along one axis only (no bending or torsion).

In the 2D model temperature does not change in the z-direction. A length of coil conductor can then be analyzed. If the cross section is divided into elements and each element is allowed to expand freely without influence from its neighbors. Fig. 3 shows the thermal expansion of each element. In the figure increasing temperature of the elements is indicated by darker color of the surfaces. The positions of the floating surfaces are exaggerated indications of the elongation of the elements. Linear thermal expansion is assumed. The free elongation can be calculated using 2.1.

$$\Delta L_{loc} = \alpha L_{ref}(T_{loc} - T_{ref}) \tag{2.1}$$

Here ΔL_{loc} is the free elongation due to the change in temperature, α is the coefficient of thermal expansion (in the case of copper $\alpha_{Cu} = 1.7\times10^{-5} 1/K$), L_{ref} is the original reference length of the finite element, and $T_{loc} - T_{ref}$ the temperature change from reference temperature to local temperature causing the elongation.

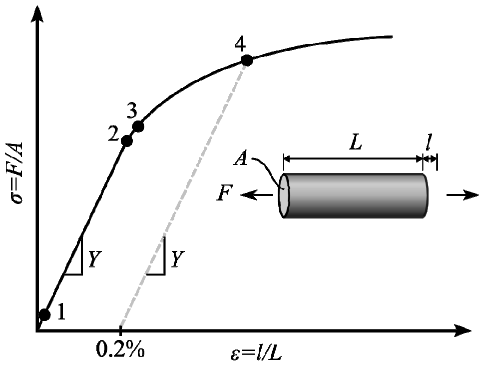

Fig. 4. Typical yield behavior for non-ferrous alloys, adapted from [9].
1: True elastic limit
2: Proportionality limit
3: Elastic limit
4: Offset yield strength

2.2. Generated Stress and Strain

If the elements were allowed to expand as shown in Fig. 3, the coil would be bent due to the elongated elements. In this system no bending is allowed, so all elements have to have the same length, meaning that the warm and long elements get compressed in pulling the colder and shorter elements. This generates strain in the elements. Tensile strain is generated in the colder elements and compressive strain in the warmer. The strain generates forces. These forces divided by the area they act on are the stresses. If temperature differences are sufficiently small, the strain is kept in the elastic region under the proportional limit (below 2 in Fig. 4).

At these small strains the stress-strain relation is linear following Hooke's law:

$$Y = \frac{stress}{strain} = \frac{\sigma}{\varepsilon} \tag{2.2}$$

Here $stress, \sigma$ is force per area, $strain, \varepsilon$ the ratio of elongation divided by original length, and Y the Young's modulus a material property, indicating the ratio between the stress and strain. (The Young's modulus for Cu, $Y_{Cu} = 1.1\times10^{11} Pa$). This linear connection between stress and strain allows the total elongation of the piece of inductor coil to be calculated as a function of the mean temperature.

$$\Delta L_{avg} = \alpha_{Cu} L_{ref}(T_{avg} - T_{ref}) \tag{2.3}$$

The new length of the elements is found by:

$$L_{avg} = \alpha_{Cu} L_{ref} (T_{avg} - T_{ref}) + L_{ref} \tag{2.4}$$

This is the length all the elements are forced to. Strain is calculated as the ratio of the change in length divided by the length at zero stress:

$$\varepsilon_{loc} = \frac{L_{avg} - L_{\sigma=0}}{L_{\sigma=0}} \tag{2.5}$$

$L_{\sigma=0}$ is a local value depending on temperature.

$$L_{\sigma=0} = \Delta L_{loc} + L_{ref} \tag{2.6}$$

Thus the local strain is given by:

$$\varepsilon_{loc} = \frac{L_{avg} - L_{\sigma=0}}{L_{\sigma=0}} = \frac{(\alpha_{Cu} L_{ref} (T_{avg} - T_{ref}) + L_{ref}) - (\alpha_{Cu} L_{ref} (T_{loc} - T_{ref}) + L_{ref})}{\alpha_{Cu} L_{ref} (T_{loc} - T_{ref}) + L_{ref}} \tag{2.7}$$

Setting $T_{ref} = T_{avg}$ and $L_{ref} = 1$, then (2.7) is reduced to:

$$\varepsilon_{loc} = \frac{-\alpha_{Cu} (T_{loc} - T_{avg})}{\alpha_{Cu} (T_{loc} - T_{avg}) + 1} \tag{2.8}$$

With the limitations described, local strain can be found by calculation from the local and mean temperature and the thermal expansion coefficient. The thermal information is known from the temperature model.

3. Fatigue

3.1. Stages of Fatigue

Fatigue is failure that occurs after cyclic stress often at much lower stress levels than the yield strength of the material, Fig. 4, point 3. Fatigue requires a sufficiently high tensile stress, large enough fluctuations in the stress, and a sufficiently large number of cycles of applied stress[5]. Fatigue develops over a large number of cycles. During this development four stages has been identified [6]:

1. Crack initiation – Early fatigue damage. Can be removed by suitable annealing.
2. Slip-band crack growth or Stage I crack growth – Deepening of the initial cracks on planes of high shear stress.
3. Crack-growth on planes of high tensile stress or Stage II crack growth – Growth in well-defined crack in direction normal to maximum tensile stress.
4. Ultimate ductile failure – occurs when crack reaches sufficient length so the remaining section cannot support the applied load.

This list is made from a mechanical structural point of view. For electrically induced temperature increase, these cracks would probably at some point, increase the losses leading to an accelerated breakdown, not by ductile failure, but more likely by hotspots leading to melting and water leakages.

3.2. Modelling Fatigue

Fatigue is generally divided into two main groups: High Cycle Fatigue (HCF) and Low Cycle Fatigue (LCF). High Cycle Fatigue occurs at stresses below yield, but requires a high number of cycles before failure (generally above 10^4 cycles). Low Cycle Fatigue occurs when stress is above general yield. Relationships are found empirically for the number of cycles a test object can take before failure. Originally developed for the low cycle fatigue region the relation called the Coffin-Manson relation has been shown to provide good results also for the HCF region [8]:

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f (2N)^c \tag{3.1}$$

where $\Delta\varepsilon_p / 2$ is the plastic strain amplitude, ε'_f the fatigue ductility coefficient (=2.18), $2N$ the number of strain reversals to failure, and c the fatigue ductility exponent (=-0.66) using annealed copper [8].

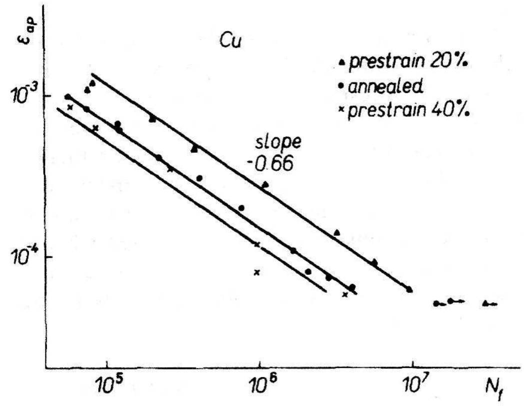

Fig. 5. Strain – fatigue life relation for Cu [8]

The relation and experimental data for the strain fatigue life relation from (3.1) is shown in Fig. 5.

These models are generally for mean stresses of zero, however in strain controlled fatigue tests this is usually relaxed after a short number of cycles and therefore neglected in this simplified models [10]. This means that the cyclic stress amplitude is divided equally between the hot and the cold state of the heat cycle. The temperature strain of the strain-model therefore has to be considered a peak-to-peak value.

4. Example

To generate estimates of fatigue life the following steps are taken:

1. The maximum and minimum peak temperature and the mean temperature are provided by the thermal model.
2. Based on the temperatures, maximum strain is found. This maximum is normally asymmetric (one-sided) and relaxed in a short number cycles, so the peak stress is half the value found by the strain model.
3. The strain is used in the fatigue model to read estimated fatigue life in number of cycles.

A 10 mm diameter circular tube with wall thickness of 1 mm carrying a current of 4.9kA at 10 kHz is considered. The cooling was water at 20 °C flowing at a rate of 35 l per minute:

1. The thermal model gives an mean temperature of 55 °C. The maximum and minimum values are 109 °C and 28 °C. The largest deviation from mean temperature is 54 °C.
2. Peak strain is found by (2.8) and as a value of $9.2x10^{-4}$. This is an asymmetric strain, so it is relaxed to half the value $4.6x10^{-4}$.
3. The strain value is entered in (3.1) or used to read cycles for fatigue from the graph in Fig. 5, both giving a fatigue life of $5x10^{5}$ cycles.

Conclusions

Temperature modeling based on heating Joule losses and cooling by fluid flow needs a coupled model. Losses and fluid flow are temperature dependent, so nonlinearities are present. The quasi two-dimensional model made through the combination of the commercial software packages Flux2D and Fluent provides temperature information of the coil conductor in accordance with experimental data. A simplified linear model was used to calculate the mechanical stress and strain. Valid results are expected for use with high cycle fatigue, but the linear limitation makes the models use for low cycle fatigue limited. Though the metallurgical aspects of metal fatigue are not fully understood, curves of fatigue life are found in the literature. A fatigue life prediction of $5x10^{5}$ cycles was found for the investigated geometry using the thermo mechanical fatigue model.

References

[1] Stuehr, W. I., Lynch, D.: *How to Improve Inductor Life, Part 2,* Proceedings of the 24th ASM Heat Treating Society Conference, September 17-19, 2007, pp 1-4

[2] Rudnev, V. I.: *Systematic analysis of induction coil failure*. Part 1 – 11. Heat Treating Progress , August 2005 – September/October 2007.

[3] Nemkov, V., Goldstein, R.: *Influence of Cooling Conditions on Induction Coil Copper Temperatures.* Proceedings of the International Symposium on Heating by Electromagnetic Sources, Padova, 2007, pp. 191-199

[4] Svendsen, K, Hagen, S. T., Melaaen, M. C.: *Temperature distribution in selected cross sections of induction heating coils*. Proceedings of the International Symposium on Heating by Electromagnetic Sources, Padova, 2007, pp. 357-364

[5] Dieter, G.E.: Mechanical Metallurgy, McGraw Hill, 1988, 375-395pp, 1988

[6] Plumbridge, W. J. and Ryder, D. A.: *Metall. Rev.*, vol. 14, no. 136, 1969

[7] Lukas, P., Kensil, M.: *Cyclic Stress-Strain Response and Fatigue Life of Metals in Low Amplitude Region*, Material Science and Engineering, Vol 11, pp 345-356, 1973

[8] Lucas, Klesnil, Polak: *High Cycle Fatigue Life of Metals,* Material Science and Engineering. Vol 15 1974 s239-245

[9] http://en.wikipedia.org/wiki/Yield_Stress

[10] Lorenzo, F., Laird, C.: *A new approach to predicting fatigue life behavior under the action of mean stresses*, Mater. Sci. Eng., Vol 62, pp. 206-210, 1984

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