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Development of Mathematical Model for Determining Sound Reduction Index of Building Elements

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Abstract

Sound can be pleasant or disturbing; the disturbing sound is called noise. Noise can cause wide variety of damage from medical to economical. House that is built next to the railroad is less valuable; workers, exposed to noise lose concentration that result in decrease of efficiency. Loud noise can cause pain and irreversible loss of hearing.

The walls of a building protect us from the traffic noise; inner walls protect us from noise in the nearby rooms also giving us more privacy. That is why materials with good sound isolation properties are needed [1]. Sound by its source can be divided in two groups: impact and airborne sound, in this research the latter is studied.

For determination of sound reduction index, standardized measurements must be carried out that are expensive and therefore good mathematical model can save a lot of money. Furthermore, if the experiment shows that the building element does not meet the necessary sound reduction index, research work is still needed to improve it.

Keywords: acoustics, sound reduction index.

1. Introduction

The main goal of this work is to develop a mathematical model that can calculate the sound reduction index of building elements without actually carrying out the measurement. To achieve the goal a real building element form A/S Lode was used (Fig.1). The dimensions of the block are 440x245x238mm. It has air gaps smartly distributed for better thermal insulation.

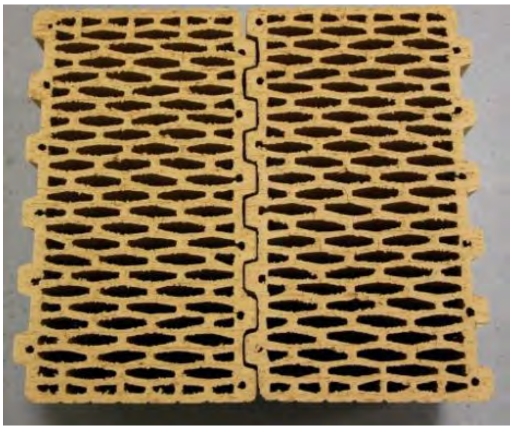

Figure 1. JSC “Lode” clay building block “keraterm 44”.

This particular block was chosen because standardized measurement has been carried out and later the model can be validated. For measurements of sound reduction index the experimental setup and specifications are found in [2][3]. The frequency range in question is 1/3 octave band from 100Hz to 3150Hz. In this work a 2D model is used, more on that in 3.3.

The problem consists of two parts, the first is to develop a model of test room where different materials can be measured and that is close to real environment; second is getting to know, what is happening inside the material being tested. This is preparatory study so only built in features will be used.

Impedance boundary conditions will be compared to sound hard boundary conditions in this work.

Boundary conditions on solid domains and test room walls will be varied.

Rayleigh damping will be used to describe the damping in solid domains.

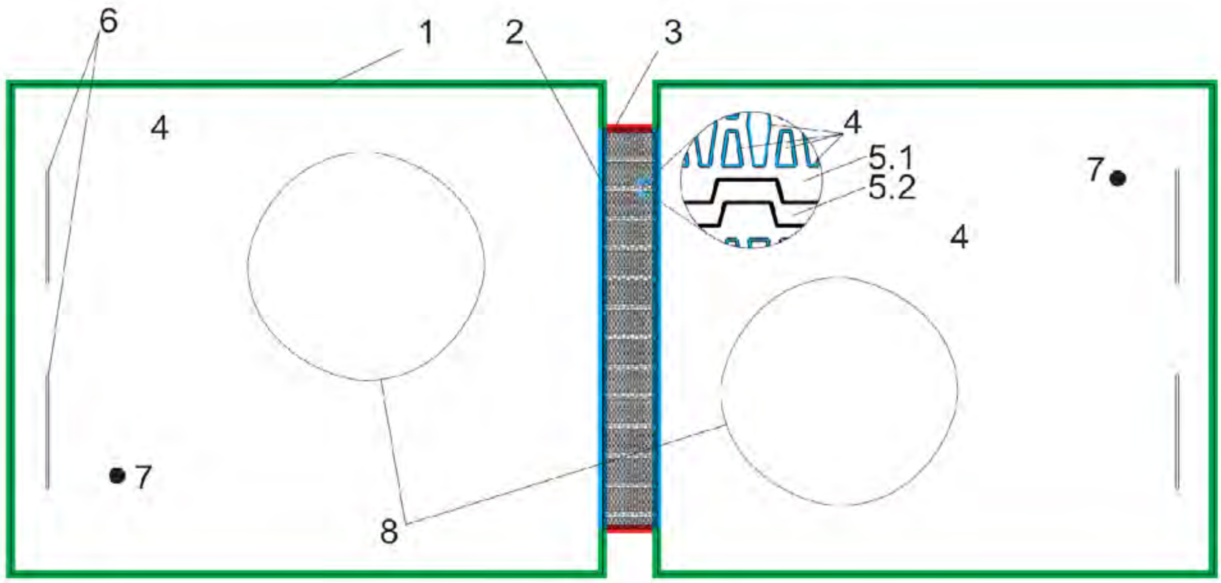

Figure 2. Model description, domains and boundary conditions.

1) Sound hard or impedance boundary conditions (green boundaries).

2) Acoustic-structure interaction boundary (gold boundaries).

3) Symmetry/fixed constraint boundary condition (red boundaries).

4) Air domains

5) Solid domains

5.1. Keraterm44

5.2. Concrete

6) Sound diffuser

7) Power point sources

8) Rotating microphone (for sound pressure level registration).

2. Governing equations and boundary conditions

Helmholtz equation:

$$\nabla\left(-\frac{1}{\rho_0}(\nabla p - q)\right) - \frac{\omega^2 p}{\rho_0 c_s^2} = Q, \tag{1}$$

where

ρ_0 – density of fluid or gas,
ω – angular frequency,
p – pressure,
c_s – speed of sound in fluid or gas,
q – dipole source,
Q – monopole source.

Boundary condition for acoustic-structure interaction are as follows:

$$\begin{cases} F_p = -n \cdot p \text{ (fluid} - \text{structure)} \\ a_n = n \cdot u_{tt} \text{ (structure} - \text{fluid)} \end{cases} \tag{2}$$

Structure acoustics:

$$\varepsilon_{mn} = \frac{1}{2}\left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m}\right), \tag{3}$$

$$s = s_0 + C:(\varepsilon - \varepsilon_0 - \alpha\theta), \tag{4}$$

where

u – displacement,
ε – stress,
s – strain,
C – 4th order elasticity tensor,
ε_0 – initial stress,
s_0 – initial strain,
α – thermal expansion tensor,
θ – temperature.

Impedance boundary condition:

$$-n\left(-\frac{1}{\rho}(\nabla p - q)\right) = -\frac{i\omega p}{Z_i}, \tag{5}$$

where Zi – impedance.

Sound hard boundary:

$$\frac{\partial p}{\partial n} = 0. \tag{6}$$

Sound pressure level equation:

$$L_p = 10 \cdot \lg\left(\frac{\sum p_{prim}^2}{\sum p_{sec}^2}\right), \tag{7}$$

where

L_p - sound reduction index,
p_{prim} – sound pressure level in primary room,
p_{sec} – sound pressure level in secondary room.

The sound reduction index is calculated by measuring the loudness of sound in first room and in the second room, than subtract second from first. By using logarithm properties equation 7 can easily be acquired. There are many more factors in the standardized measurements with sound sideway effects, but they do not occur in mathematical model.

Reyleigh damping parameters:

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\omega_1} & \omega_1 \\ \frac{1}{\omega_2} & \omega_2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \tag{8}$$

where

α,β – Rayleigh damping parameters,
ω_n – natural frequencies,
ξ_n – damping factor.

3. Modelling methodology

3.1. Model setup

To solve the problem Comsol Multiphysics Acoustic-Structure interaction module was used. In the air domains equation 1 was used. The acoustic pressure loses in air was neglected as they are of no significant value. In the solid domains equations for solid dynamics were used (3) and (4).

As a source the intensity point sources was used with a value of 0.01W that is 2Pa of pressure amplitude or 100dB loud sound. This value did not matter because sound pressure level is calculated as pressure in primary room with respect to pressure in secondary room (7). In one case the stationary wave field was achieved with source in the first room; in the other case the source was active in secondary room.

The two circles in Fig. 2 is place where the rotating microphone registers pressure level. The pressure is integrated for full circle.

Full frequency spectrum is used in the standardized measurement, unfortunately frequency domain do not allow to generate noise, so 1/27 octave band frequencies were chosen.

The microstructure of building block in question has a capillary microstructure. However, this is an important factor we did not take into account in this study.

The Rayleigh damping was included. Values α and β were calculated using equation (8). This included calculation of natural frequencies of the wall, which were also computed using Comsol Multiphysics, but is not included in this study. Natural frequencies and Rayleigh damping coefficients can be seen in Fig. 3. Only the first two natural frequencies were used for computation of damping parameters.

Table 1: Natural frequencies of the test wall and Rayleigh damping coefficients.

freq\BC	fixed	symmetry
f_1	108,7	36,49
f_2	171,9	100,38
f_3	235,35	165,16
f_4	298,93	228,74
Damping factor: 0,05		
α	41,84	16,81
β	5,67E-05	1,16E-04

3.2. Boundary conditions

Eight calculations were carried out with different boundary conditions each. On the boundaries between test room and surrounding space an impedance boundary condition was used. The impedance boundary condition was chosen because it the best describe surrounding space. The alternative was to use sound hard boundary (6) which is easier to solve for. This dual approach was made because there are not any specifications about test room walls. So it is important to know whether those boundary conditions affect the final result and by how much. It is important to set up the same conditions as in the test room for validation of the results.

It was difficult to find boundary conditions for the red boundaries (see Fig. 1) that suits the physical reality. This boundary was treated as fixed in one case and symmetrical on the other. Neither seems to be good, at least they represent extreme cases. This is one of the fields in which an improvement is necessary.

3.3. Meshing

A problem for this study is the wavelength resolution. The highest frequency solved for was 3582.1Hz that has the wavelength of 0.096m. It is advised to use at least 5 degrees of freedom (DOF) per wavelength to achieve meaningful solution. This led to largest element size of 0.019m. The 3D geometry has too DOFs to calculate the result so a 2D model was set up. It has a drawbacks discussed later. Although it is advised for completely reliable solution at least 10 DOFs per wavelength, for higher frequencies it was not achieved.

4. Results

The value of sound pressure was calculated. Pictures of sound pressure level in dB distributed in primary and secondary room with source in secondary room. It can be seen in zoomed in pictures that sound in brick’s air enclosures is higher attenuated (Figs. 4, 6). In (Figs. 3, 5) is shown that pressure level field is dependent on frequency.

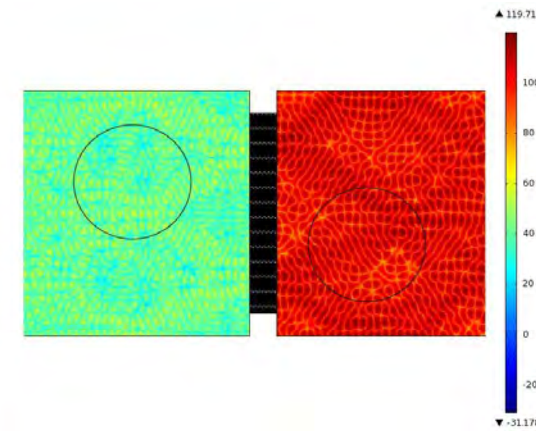

Figure 3. Sound pressure level in dB at 1500Hz.

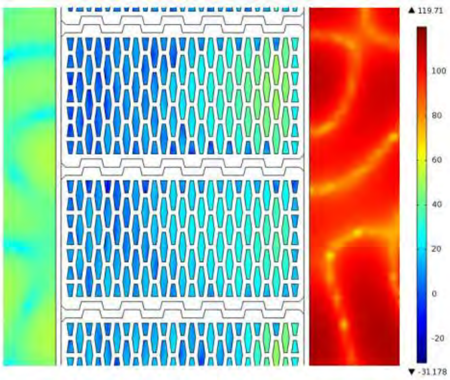

Figure 4. Sound pressure level in air enclosures at 1500Hz.

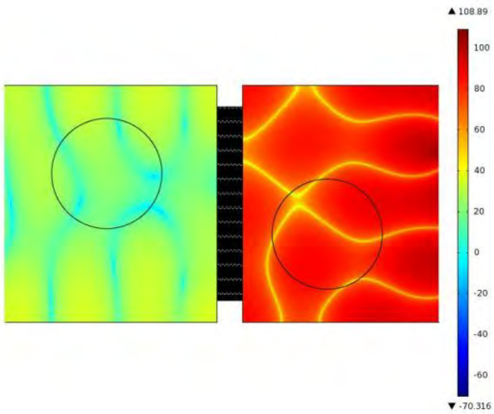

Figure 5. Sound pressure level in dB at 150Hz.

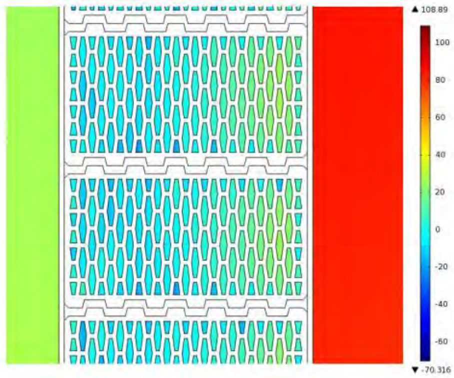

Figure 6. Sound pressure level in air enclosures at 150Hz.

To achieve the result the computed pressure amplitudes values were put into equation (7) and values for 1/3 octave band frequencies were computed by using algorithm specified in [3] the single value sound reduction index was calculated being shown in Tables 2 and 3.

Table 2: Single value sound reduction index with source in primary room.

	symmetry	fixed constraint
Impedance	52	47
Hard bound.	52	47

Table 3: Single value sound reduction index with source in secondary room.

	symmetry	fixed constraint
Impedance	51	47
Hard bound.	51	47

As shown in tables 2 and 3 the values for sound hard and impedance boundaries give the same result. The comparison between point sources in each room with the same boundary conditions (fixed wall and sound hard boundary) can be seen in Fig. 7. No one match the theoretical values well. There are differences between primary and secondary source.

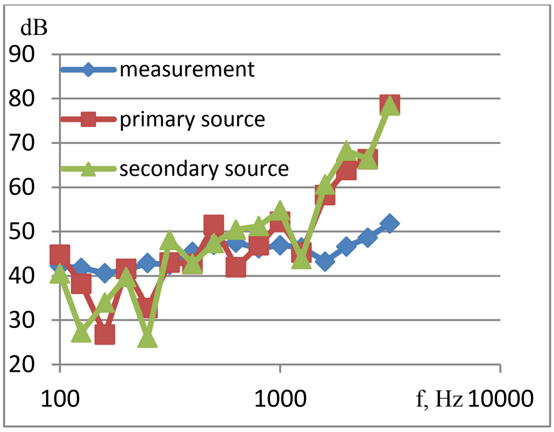

Figure 7. Sound reduction index as function of frequency with source in different rooms.

The experimental value for the same wall was 47dB with 1/3 octave band frequency values shown in figure 3 for comparison.

Mesh selection seems to be sufficient for this study as the 5 DOFs per wavelength was the maximum element size; however the greatest result difference with finer mesh was 5.2%. For higher precision results an improvement in mesh is still necessary.

5. Discussion

It is clearly seen that calculated values do not show good fit with experimental ones. This means that improvements must be made. The drawbacks of the particular model are:

- Natural frequencies of the wall in 2D geometry are different from those of real wall in 3D case that can affect the result.
- Rayleigh damping gives good values only for a few modes. That is why another damping model must be implemented in Comsol Multiphysics for this wide frequency range.
- Boundary conditions between model wall ends and test room walls must be considered, because given ones do not seem to be physically correct.

However, there is still a good chance that model can be developed:

- L_p values calculated in this simple model is of the same magnitude as the experimental and have similar shape, except for high frequencies. The mismatch can be of two reasons. First, the high frequency range is less accurate. Second, the Rayleigh damping must have played a part because the value of β in high frequency range become important.
- Another important theoretical match is the shape of wall deformation (fig.8). It agrees with the theory of sound transmission through walls by bending them. [1][5]

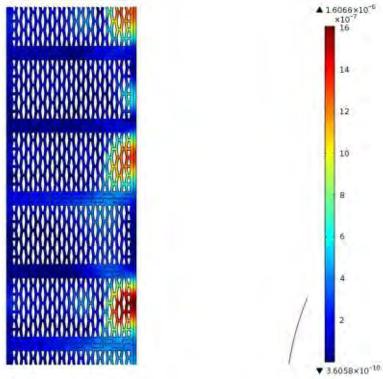

Figure 8. Wall deformation at 1400Hz.

When switching the point source from one room to other a significant result difference can be seen. It is due to geometry of room and its resonance frequencies. Therefore sound diffusers must be used to reduce this effect.

6. Conclusion

Although the desired result was not achieved, it still gives a good chance that one can be developed using Comsol Multiphysics.

This particular work show that there are many fields how to improve the model. First is to develop sound diffusers to acquire homogenous sound field in the test rooms, so that accidental values of sound pressure field at pressure level registration place did not give practically unrealistic value. Similar problem of diffusing sound is studied in [6].

The impedance boundary conditions give almost the same sound pressure level values as sound hard boundary does, so in the future the sound hard boundaries can be used to reduce computational costs.

The porosity must be included in model because it gives better physical approximation of the real system and Comsol Multiphysics poroelastic material model promises to be the right solution of this challenge.

7. References

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8. Acknowledgements

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