STEEL QUenchING PROCESS AS HYPERBOLIC HEAT EQUATION FOR CYLINDER

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Intensive quenching method in water in opposition to oil is offered last decade [1]. To describe quenching process by hyperbolic heat exchange equation [2]-[4] in contradiction to classic heat exchange equation by co-author was offered. From practical point of view there is no possibility to determine the velocity of heat flux at initial moment of process.

Time invert hyperbolic heat exchange problem for cylinder is discussed in this paper. Definition of problem in domain \( r \in (0, R), z \in (0, H), t \in (0, T), \varphi \in [0, 2\pi] \) is as follows:

\[
\tau_r \frac{\partial^2 U}{\partial t^2} + \frac{\partial U}{\partial t} = a^2 \left[ r^{-1} \frac{\partial}{\partial r} \left( r \frac{\partial U}{\partial r} \right) + \frac{\partial^2 U}{\partial z^2} + r^{-2} \frac{\partial^2 U}{\partial \varphi^2} \right].
\]

(1)

Additional following boundary conditions when \( t \in [0, T], \varphi \in [0, 2\pi] \) are formulated:

\[
\frac{\partial U}{\partial r} + k_r U = \gamma_1(z, \varphi, t), \quad r = R, \quad z \in [0, H],
\]

(2)

\[
\frac{\partial U}{\partial z} + k_z U = \gamma_2(r, \varphi, t), \quad r \in [0, R], \quad z = H,
\]

(3)

\[
\frac{\partial U}{\partial z} - k_z U = \gamma_3(r, \varphi, t), \quad r \in [0, R], \quad z = 0.
\]

(4)

Periodical boundary conditions when \( r \in (0, R), z \in (0, H), t \in (0, T) \) are following:

\[
U(r, z, 0, t) = U(r, z, 2\pi, t), \quad \frac{\partial U(r, z, 0, t)}{\partial t} = \frac{\partial U(r, z, 2\pi, t)}{\partial t},
\]

(5)

Additional condition is following:

\[
\frac{\partial U}{\partial r} \to 0, \quad r \to 0, \quad z \in [0, Z], \quad t \in [0, T].
\]

(6)

Usually two following initial conditions are formulated:

\[
U = U^0(r, z, \varphi), t = 0, \quad r \in [0, R], \quad z \in [0, H], \quad \varphi \in [0, 2\pi],
\]

(7)

\[
\frac{\partial U}{\partial t} = V_o(r, z), t = 0, \quad r \in [0, R], \quad z \in [0, H].
\]

(8)

Value \( V_o(r, z) \) of initial condition (8) usually is unknown. Therefore two conditions at the moment \( t = T \) of process are given instead of initial condition (8):

\[
U(r, z, \varphi, T) = U_T(r, z, \varphi), \quad \frac{\partial U}{\partial t}(r, z, \varphi, T) = V_T(r, z, \varphi)
\]

(9)

It is shown that solution of problem (1)-(7), (9) can be found as solution of first or second kind Fredholm’s integral equation. Several particular cases are discussed: a) thin disk (cylinder with small height); b) thin cylinder.

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REFERENCES