Mathematical Modelling of a Living-Room with a Solar Radiation Source

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Introduction

Heat balance in a living-room is very important from the energy consumption angle and available solar radiation has an essential influence on it. For the first approximation this kind of heat transfer can be neglected and only heat conductivity in the boundary constructions and convection in the room can be observed. The radiation heat transfer from the heater's surface and through the window must be taken into account for more accurate numerical results. On sunny days, owing to the considerable solar heat transfer through the window, required temperature in the room can be maintained with less heat from the central heating system. The mathematical modelling enables us to include the solar heat source and radiation heat transfer from the heater in the total heat consumption. Another important issue to be taken into account is thermal comfort conditions. They are generally affected by many factors, such as velocity of airflows, humidity, absolute temperature and temperature gradient in the room. It is therefore necessary to analyse these factors using different variants of model conditions.

Problem Formulation

A living-room with convective boundary conditions on exterior border structures and heater surface temperature is modelled. A radiation heat transfer model with an additional solar source is also included. This approach characterises the effect of the radiation heat transfer and the conditions of thermal comfort. *ANSYS CFX* software is used for development of 3D models and numerical calculations. The calculations have been performed for a room shown in Fig.1. Heat transmittance is set for the window U=2.5 W m^{-2} K^{-1}, for the exterior wall U=0.35 W m^{-2} K^{-1}. Heat transmittance for other boundaries is set to 1.0 W m^{-2} K^{-1}. The heat transfer coefficient from the surfaces is included in the total heat transmittance value. It is assumed that the surrounding rooms have the temperature of 20 ºC, the corridor - of 15 ºC. The outdoor temperature is set as for winter conditions (T=−10 ºC). Solar radiation through the window is modelled as a heat source on the inner window's surface with heat flux density 500 W/m^2 and variable angle of attack from 30 to 60 degrees, which models different sun altitudes. For the radiation simulation the Monte Carlo model is used and all objects except the window are modelled as grey bodies with emission ε=0.9.

To describe the quasi-stationary behaviour of temperature and average turbulent flows traditional differential equations and SST $k-\omega$ turbulence model are employed. 3D discretisation was performed with tetrahedral and smaller prismatic elements of varying size from 20 cm to 0.5 mm in the vicinity of the heater. The total number of elements reaches $7\cdot10^5$ depending on geometry. The example of typical mesh near the heater is shown in Fig. 2. The total time required for one variant calculation with a 3 GHz computer is about 2 days. The calculated heat imbalance between the heater power, the solar source and the losses from the outer surfaces decreases below 3 %.

Different features of considered variants are summarized in Table 1. The variant A represents a room with radiation from the heater's surface, but without solar heat source. The angle of solar radiation is varied in variants B1-B3. Variant C represents a case with a solar source, but without heat transfer from the heater (adiabatic condition), which represents a room in winter conditions with a switched-off heating system.

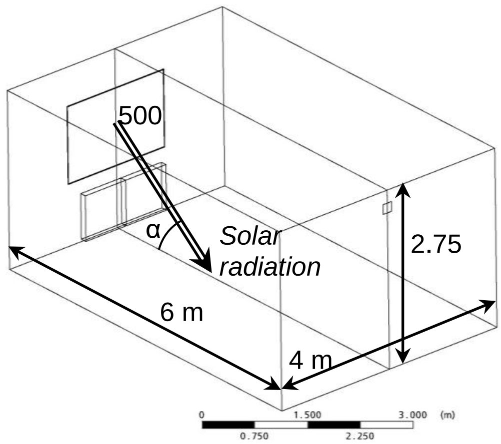

Fig. 1. Layout of a modelled room.

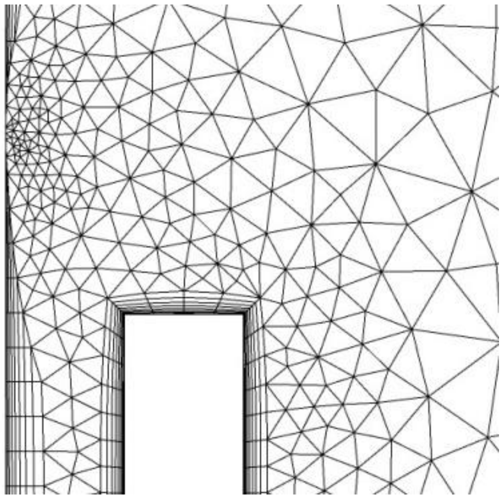

Fig. 2. Typical mesh near the heater.

Table 1. Properties of various variants considered and main results of the modelling.

Properties/results	Variants (variable parameters are bold)				
	A	B1	B2	B3	C
Angle of attack α (degrees)	-	**30**	**45**	**60**	45
Boundary condition on heater	temperature, 50°C				**adiabatic**
Heat amount for the heater (W)	225	173	173	178	0
Solar power W	0	411	327	228	333
Average velocity *v* (cm/s)	2	5	6	5	3
Average temperature *T* (°C)	24.2	32.5	30.8	28.4	26.0
Vertical temperature difference Δ*T* (°C)	2.1	1.7	1.7	2.5	1.5

Results

Results for all considered variants are summarized in Table 1 and visualized in Fig. 3. It is possible to extract two significant parts of the results – heat balance of the room (heating and solar powers) and thermal comfort conditions (average velocity and temperature, vertical temperature difference).

For the basic variant without a solar source an average temperature in the room is 24.2°C and there is a characteristic hot air flow at the top of the heater near the window (Fig. 4). Results of the variants B1-B3 with solar sources show that at the small angles of attack the solar radiation power is increased because the sun is shining deeper into the room. Thus, the average temperature in variant B1 with a 60° angle of solar source is 28.4°C, but in variant B3 with a 30° angle it is 32.5°C. Thermal comfort conditions in this situation are not suitable and the thermostatic control of the heating system or installation of a ventilation system is necessary to decrease the temperature. Results for variant C with a switched-off heating system show that the power of solar radiation provides sufficient heat amount for the average temperature of 26 °C and additional heating is not needed. This result is achieved for the room without air infiltration and shows the maximum potential temperature in the room. Convection heat transfer is to be taken into account to model rooms more accurately.

Thermal conditions in the room are connected to the structure of airflow velocities, temperature distribution and thermal convection. In the case of solar radiation, hot regions on the floor produce sizeable convection and increase average velocities in the room. In general, the radiation source through the window increases airflow intensity in the middle part of the room (Figs. 3, 4).

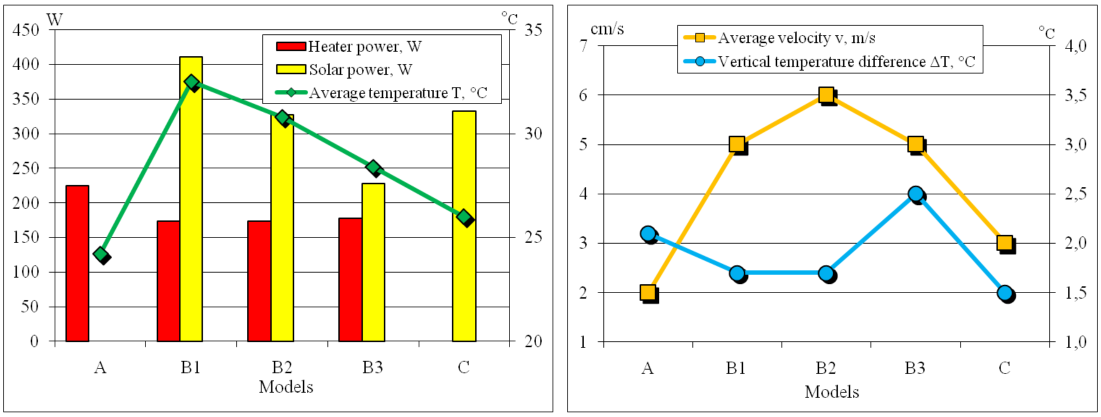

Fig. 3. Heating powers, average temperature, vertical temperature difference and airflow velocity in the middle of the room for different variants.

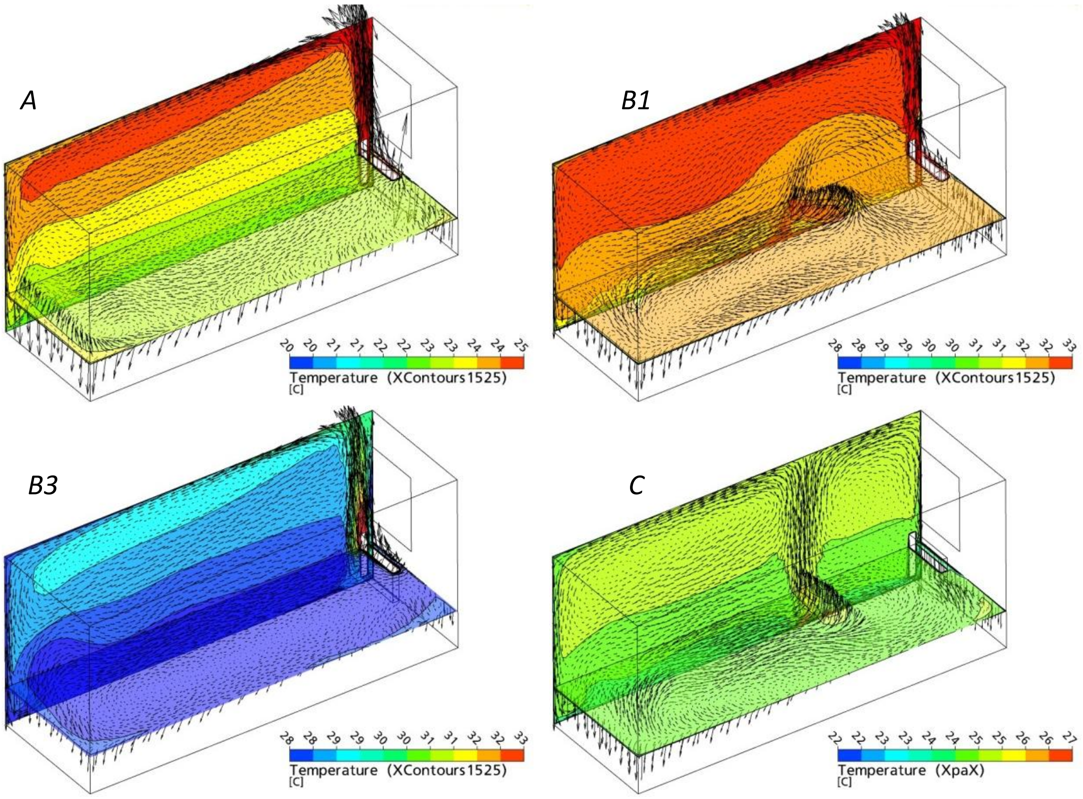

Fig. 4. Velocity vector field and temperature contours in middle vertical cross-section and 0.5m height horizontal cross-section for variants A, B1, B3, C.

Conclusion

The developed model allows estimation of temperature, airflow distribution and the tendencies of its changes, as well as a dependence on heat loss through the boundary structures under different solar radiation conditions in living-room. Obtained results show the essential influence of some variable factors on the heat balance and the thermal comfort conditions. 3D modelling of a living-room including a solar heat source is very important for the correct representation of qualitative and quantitative heat transfer and convection processes in the living-room. The best conditions for the modelled room with solar radiation are in the following cases:

- the heater's power is being regulated in accordance with the solar radiation intensity and indoor temperature;
- temperature difference in the adjoining rooms is as small as possible;
- the angle of solar radiation attack is >45°.

If the above-mentioned conditions are fulfilled, intensive airflows will not be formed and an average temperature in the room will be acceptable for thermal comfort conditions.