Mathematical Modelling of Living Room with Different Types of Heating and Pressure Conditions

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Abstract: The paper deals with temperature and average turbulent airflows distributions in living rooms in a 3D approximation using Ansys/CFX software. The heat balance of a room and its dependence on various geometrical and physical factors are also considered. As physical parameters of thermal comfort conditions, the airflow velocities and indoor temperatures with its gradients are analysed. The distributions are calculated depending on the placement of the heater, the type of heating (convector or floor heating), and the pressure difference between opposite walls. The influence of these factors on the air circulation and temperature field, as well as the related heat flows through building structures is analysed. It is shown that it is possible to reduce the heating power and maintain the conditions of thermal comfort in the room at the same time. In addition, the optimal heater location and the best type of room-heating are discussed from different points of view in case of various ventilation conditions.

Key-Words: Mathematical Modelling, Living Room, Thermal Comfort, Heater, Floor Heating, Heating Power, Pressure Difference.

1 Introduction

Personal feeling of comfort is generally affected by many objective and subjective factors [1]. Physical parameters like velocity of airflows, absolute temperature and amplitude of the vertical temperature gradient in the room are very important for providing an optimal thermal comfort conditions, thus it is necessary to analyse these factors in different models of living rooms.

Air exchange is very important for rooms inhabited by humans in order to guarantee oxygen feeding, therefore airflows through openings and ventilation system are to be analysed in models with different pressure conditions. However, the greater air exchange rate means not only more fresh air, but also greater convective heat losses and increase of heating amount.

Optimal arrangement of the heater and appropriate installation of controllable venting system allows maintaining thermal comfort in a living room with reduced heat consumption. A physical model of heat balance for a living room with various physical conditions and different geometries is used, which allows analysing the distributions of the airflows and temperature. The mathematical modelling enables to choose the optimal type of heating and placement of the convector in case of different pressure conditions, in order to decrease the heat losses and improve the conditions of thermal comfort.

2 Problem Formulation

A room with different boundary conditions (convection, surface temperature, air openings) is modelled to help understanding the features of heat transfer process in the room as well as distribution of various characteristic quantities and their dependence on the different geometrical and physical conditions. A placement of the heating element (convector) varies, a model with the floor heating is developed, and their influence on the distributions of temperature and velocity fields is analysed under different pressure conditions, characterising the conditions of the thermal comfort. Ansys/CFX software is used for developing 3D mathematical models and numerical calculations.

The calculations have been performed for the room shown in Fig.1, filled with air. The window and the wall to the exterior air are modelled using different materials with heat transmittance U for the window 2.5 W/(m^2·K) and for the wall –

0.35 W/(m^2·K). Such values are chosen similar to the room with a well-insulated outer wall and ordinary double-glazed window. Between the window and the wall, a small cranny is created to model real gaps in old window-frames, however, in the opposite wall, there is a ventilation opening.

On the outer boundaries, convection boundary conditions are set with according surface heat-transfer coefficients. It is conditionally assumed that the surrounding rooms (upstairs, downstairs and side rooms) have the comfortable temperature of 20 ºC, but the end wall is contiguous with a corridor or a staircase where the temperature is lower (15 ºC). The outdoor temperature is chosen corresponding to the winter conditions (-10 ºC).

On the surfaces of crannies in the window-frame and on the ventilation opening boundary conditions with constant pressure and accordant temperature of -10 ºC and 15 ºC are defined. Pressure difference ΔP between opposite walls is set to constant 0, -1 or 1 Pa to model underpressure and overpressure in the room in cases of different windy conditions.

Four developed models with ΔP=0 and different heating options are as follows:

1 – convector placed near exterior wall;
2 – convector placed near wall to the corridor;
3 – convector placed near side wall;
4 – floor heating (without convector).

For models 2 and 4, three additional calculation series are performed with different boundary conditions on openings as follows:

2o – model 2 with ΔP=1 Pa (overpressure);
2u – model 2 with ΔP=-1 Pa (underpressure);
4o – model 4 with ΔP=1 Pa (overpressure);
4u – model 4 with ΔP=-1 Pa (underpressure).

Surface temperature of the heater is set to constant 50 ºC for variants with convector heating and to 25 ºC for models with floor heating. For all surfaces, except openings, non-slip boundary conditions are used.

In this problem formulation, the airflow in the room depends both on the convection created by the temperature difference and on the air exchange between the openings in the structures. To describe the quasi-stationary behaviour of temperature and average turbulent flows, traditional differential equations are employed [2]:

• Reynolds averaged momentum equation;
• continuity equation;
• equations for specific turbulence energy k and dissipation rate of this energy ε;
• energy conservation equation.

The turbulent viscosity is calculated using the k-ε turbulence model.

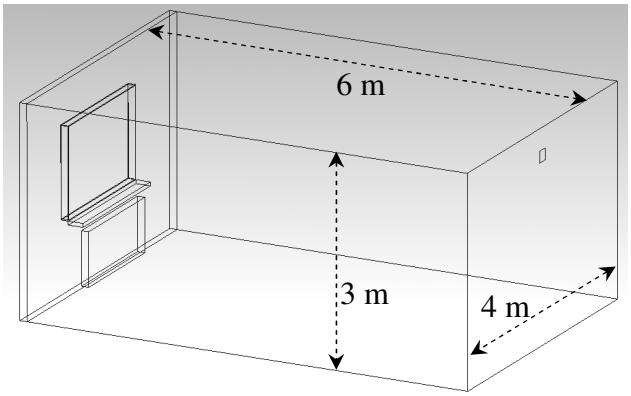

Fig. 1. Layout of a modelled room

The discretisation was performed with tetrahedral elements of varying size; boundary layers are discretised with smaller prismatic elements. The characteristic size of finite elements is from 10 cm in the middle of the room to 0.3 mm in the vicinity of the heating element and for the openings in the walls. Therefore, the total number of elements reaches $5 \cdot 10^6$ depending on geometry. The boundary conditions of the third type (convection from walls) and the low viscosity of air essentially worsen the convergence of an iteration process. The time required for calculations with a 3 GHz computer is about 5 days. The calculated difference between the heat amount from the heater and the losses from the outer surfaces and openings decreases below 5 %.

3 Problem Solution

Figures 2 and 3 show characteristic velocity fields and temperature contours from 10 to 20 ºC for considered models without pressure difference and Figures 4 and 5 – for models with overpressure and underpressure in a room with the convector and floor heating. The main results for those models are also summarised in Table 1 and visualized in Figure 6 – these can be divided into two significant groups for detailed analysis:

• heat balance of the room – heating power needed for the temperature maintenance Q (W) and the air exchange rate n (1/h) connected with the convective heat losses through the openings in building envelope;

• thermal comfort conditions – average velocity v (cm/s), mean temperature T (°C), as well as vertical and horizontal temperature differences ΔT (°C).

It is convenient to analyse each of these result groups separately, in order to choose the best room model from the viewpoint of energy consumption and with better thermal comfort conditions. As one can see from the results, it is very difficult to satisfy both of these requirements at the same time.

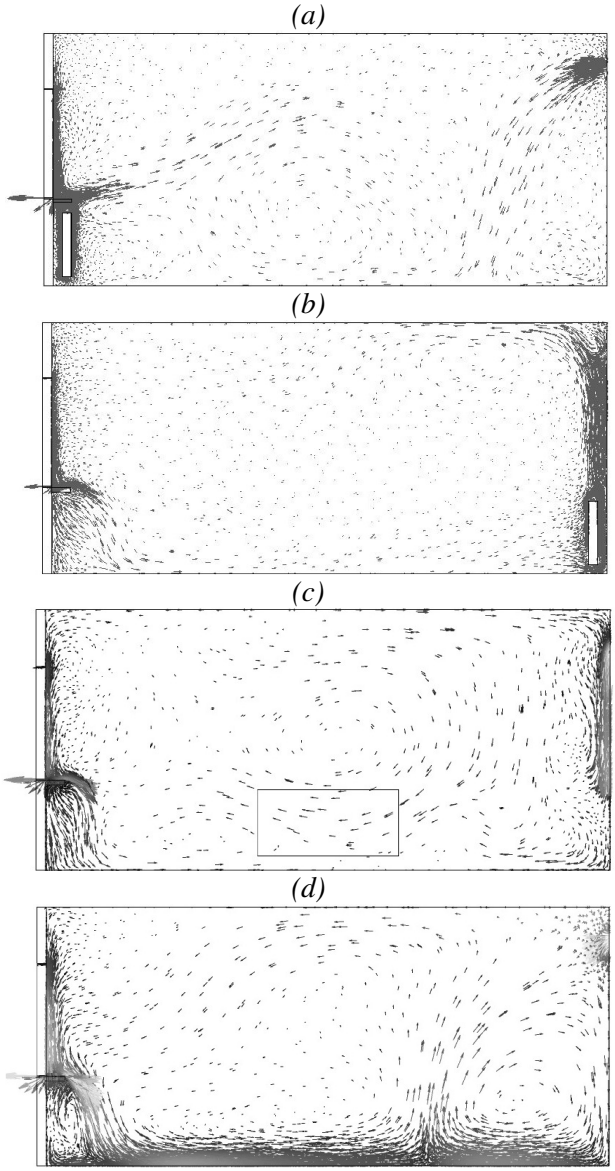

Fig. 2. Characteristic velocity vector field for models 1 (a), 2 (b), 3 (c) and 4 (d)

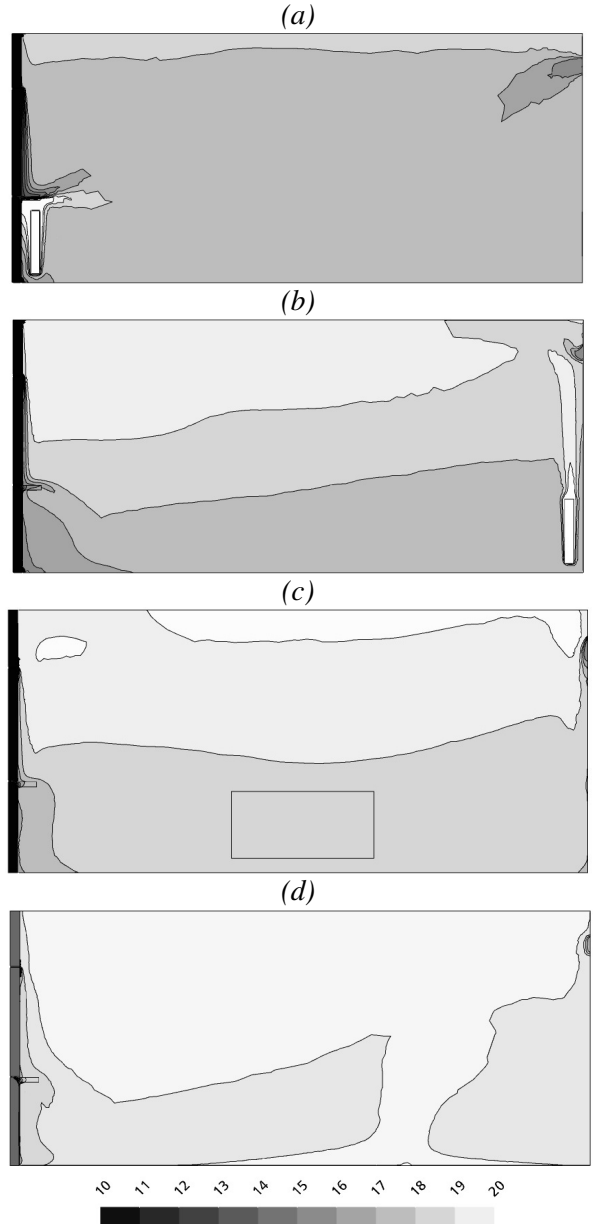

Fig. 3. Temperature contours from 10 to 20 ºC for models 1 (a), 2 (b), 3 (c) and 4 (d)

Table 1. Geometrical properties and the calculation results for different developed models

Model	Placement of the heater	Total heating power Q (W)	Air exchange rate n (1/h)	Average velocity v (cm/s)	Average temperature T (°C)	Vertical* temperature difference ΔT (°C)	Horizontal* temperature difference ΔT (°C)
1	W	164	0.5	5	17.6	0.7	6.4
2	C	138	0.15	2	18.3	2.0	4.6
2o	C	142	1.5	4	18.3	0.5	4.3
2u	C	189	1.0	7	10.2	3.9	8.6
3	S	145	0.14	4	19.2	1.9	2.8
4	F	154	0.20	3	19.0	0.8	2.5
4o	F	243	1.5	6	17.3	0.7	1.2
4u	F	542	1.1	6	14.7	5.2	5.7

W – near window, C – near opposite wall (to the corridor), S – near side wall, F – floor heating
* – in the middle of the room

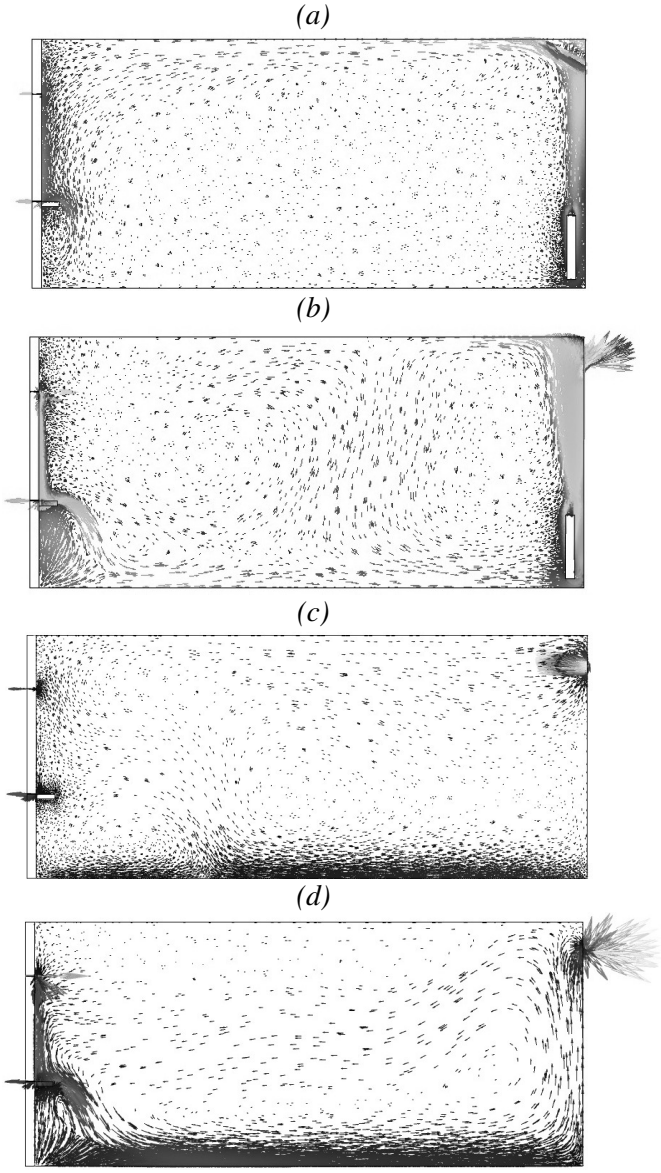

Fig. 4. Characteristic velocity vector field for models 2o (a), 2u (b), 4o (c) and 4u (d)

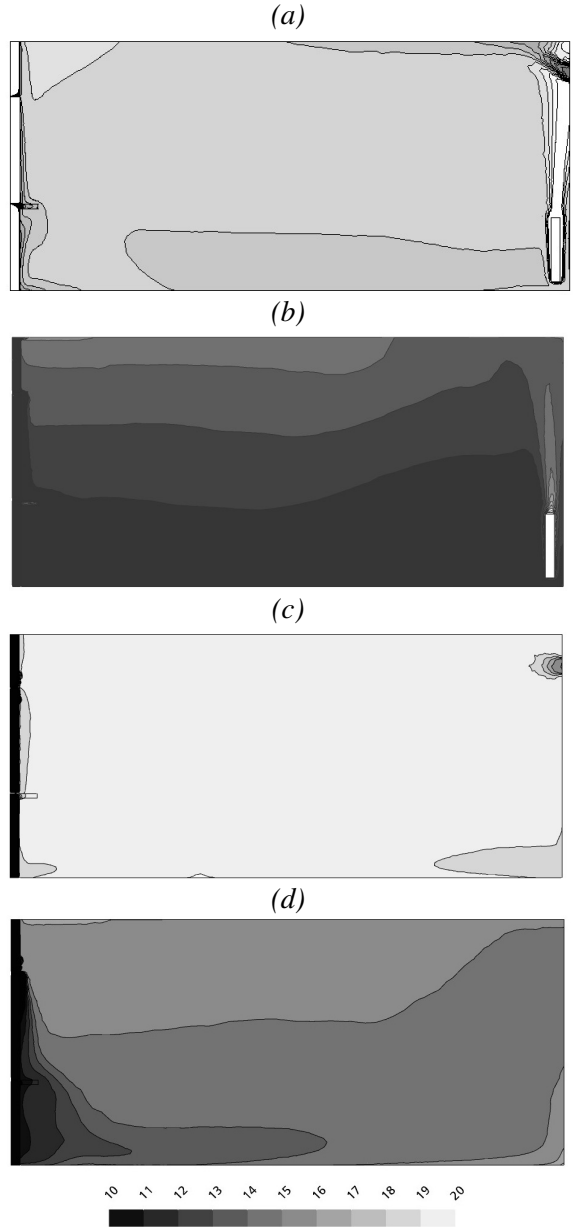

Fig. 5. Temperature contours from 10 to 20 ºC for models 2o (a), 2u (b), 4o (c) and 4u (d)

3.1 Heat Balance Analysis

Convective heat transfer from the convector is essentially dependent on the air flow intensity near its surface, and despite its constant temperature, the maximum heat is taken off when a heavy air motion occurs along it – for model 2u with underpressure in the room up to 20 cm/s near the convector and about 7 cm/s in the middle of the room (Table 1 and Fig. 4b). However, in the same model without pressure difference (model 2), the heating power is only 73% of the above mentioned – see Table 1 and Fig. 2b.

Also the floor heating is not the optimal solution from the viewpoint of heat consumption due to its large warming area – the heating power for this type of heating in model 4 without pressure difference is the same as for models with convector heating (Table 1 and Fig. 6). But due to greater electricity costs, the floor heating expenses will also be greater than for cases with central hot water heating.

Considerable increase of the heating amount for room with floor heating is observed for models with pressure difference between opposite walls – in case of overpressure (model 4o), heat transfer from the floor grows up to 157%, but in case of underpressure and, hence, cold air inflows from outside (model 4u) – to 350% in comparison with the floor-heated room without pressure difference (see Table 1 and Fig. 6).

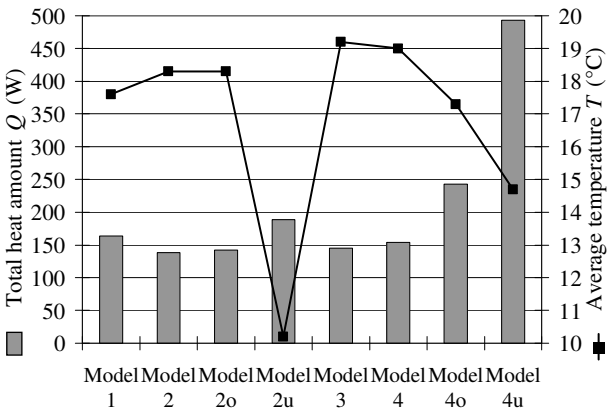

Fig. 6. Total heating power and characteristic average temperature in the middle of the room for different models

Compared together are rooms with different types of heating and the placement of a heater; one can see from the Figure 6 that in models 1, 2, 3, and 4 without the pressure difference, the heating amount is approximately equal, however pressure changes outside can increase it by more than three times (for model 4u). Hence, another significant factor related to the heat losses is the air exchange rate characterising convection through openings in the room's boundary structures. However, we cannot simply close the openings, which are necessary for maintaining the content of oxygen inhaled by people.

The air exchange rate n=0.7 (1/h) is accepted as the normal value characterising air exchange intensity in the rooms without forced ventilation. Taking into account the air inflows and outflows through slots in the window-frame and through the ventilation opening, it is obvious that from all models without pressure difference only in model 1 the air exchange is nearly sufficient due to intensive airflows near the window-sill (see Table 1 and Fig. 2a). In turn, an air exchange below the normal (models 2, 3 and 4) would decrease heat energy losses, without making people feel better at the same time – a ventilation system or natural airflows due to the pressure difference between exterior and opposite walls are needed for these rooms (see Table 1 and Fig. 2b, 2c, 2d).

Adding 1 Pa pressure difference on the outdoor openings changes airflow field in the room and supplemental heat losses through air gaps arise, especially in case of underpressure, when cold outside air comes into the room and warm indoor air flows out through the ventilation opening (Figs. 4b and 4d). For example, in model 2 with convector heating overpressure practically does not change the total heat amount, but outdoor air inflow in underpressure cases increases the total heat losses by 37%. More significant pressure difference influence on heating amount is established for the room with the floor heating (model 4o and 4u), due to large heated area (see Table 1 and Figs. 4, 6). For all models with non-zero set pressure difference, the air exchange rate in the room is between 1 and 1.5 (1/h), which explains significant heat losses by convection.

Therefore, by analysing heat losses and average temperatures in the room for different models (Table 1 and Fig. 6), one can conclude that heat losses in all rooms with zero pressure difference are similar. At the same time, model 3 will provide the highest average temperature in the room (above 19 ºC), however model 1 is the most disadvantageous from the viewpoint of heating energy consumption. However, analysis of models with different pressure conditions shows that in case of underpressure cold outdoor air inflow significantly increases heat losses from the room (up to several times) and decreases average temperature in it.

3.2 Thermal Comfort Analysis

Vertical temperature difference and average air velocities are two important factors used for human thermal comfort conditions calculations [1]. Visual comparison of those parameters for developed models of a living room is shown on Fig. 7.

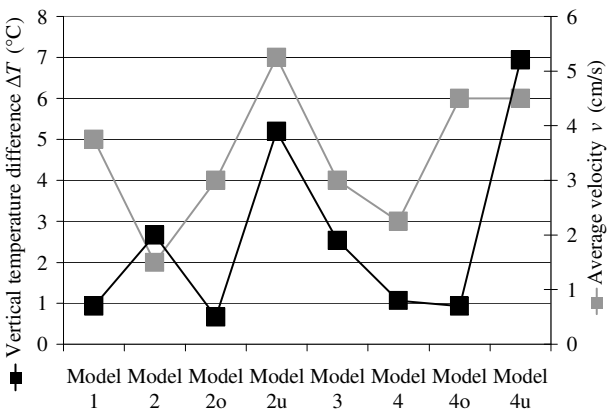

Fig. 7. Characteristic vertical temperature difference and airflow velocity in the middle of the room for different models

As the first aspect the temperature difference in the vertical direction is analysed; it should be as small as possible, but not greater than 2 ºC [1]. In case without pressure difference in models 2 and 3 (with a heater placed near the wall to the corridor and side wall accordingly), air stratification with the

vertical temperature difference in the middle of the room about 2 ºC is observed, but in models 1 and 4 temperature difference is below 1 ºC. In the first case such small temperature gradient is created due to essential air circulation in the whole room (see Table 1, Figs. 2, 3 and 7). Hot air uprising from the convector for the model 3 with great temperature stratification is shown on Figure 8.

In model 1, two considerable airflows exist near exterior opposite walls, hence temperature changes in the middle of the room are minimal – only 0.7 ºC. However, temperature fluctuations near the outer wall are notable: this is caused by active cold and hot air flows, which are partially separated by a windowsill and directed horizontally (Figs. 2a, 3a). When the air warmed by the heater is moving along its surface upward, it meets an obstacle – a windowsill, therefore the direction of the hot air stream is changed. However, at the opposite wall of the room, there is a downward inflow of cooler air through the ventilation opening. Thus, lower absolute temperature for rooms without pressure difference is observed for this model.

An interesting result is obtained for model 2u with overpressure, when there is a relatively warm air inflow trough the ventilation opening – temperature field in the room becomes more homogeneous and temperature difference reduces by four till 0.5 ºC in comparison with model 2. At the same time, the underpressure for this room increases the difference by two (see Table 1 and Figs. 3b, 5a, 5b, 7). However, for the room with the floor heating (model 4), overpressure conditions does not noticeably change temperature difference, but the underpressure increases the difference by more than six times (see Table 1 and Figs. 3d, 5c, 5d, 7).

Figure 9 shows visualisation of the temperature distribution in the room from 18 to 20 ºC and an isosurface of the 18 ºC temperature front for zero pressure difference models with side-placed convector and for floor heating. One can see there that for model 3, significant air temperature stratification is established, while for model 4 this stratification is inconsiderable. In both models, cold air inflow through opening in the window-frame near the windowsill is analogous, but in the first case, a cold airflow from the outside is present also near the wall opposite to the convector (Fig. 9a).

On the other hand, situation is symmetrical for the floor heating (Fig. 9b); cold outdoor air inflow through crannies in the window-frame in this room in case of 1 Pa underpressure is shown in Figure 10. Use of floor heating reduces vortex created by the convector with high temperature. Temperature profile in the room without pressure difference is rather vertical and its amplitude is below 1 ºC (Fig. 11b), it is coinciding well with the results of other investigations of floor heating problems [4], also in horizontal direction oscillations is below 3º C (see Table 1).

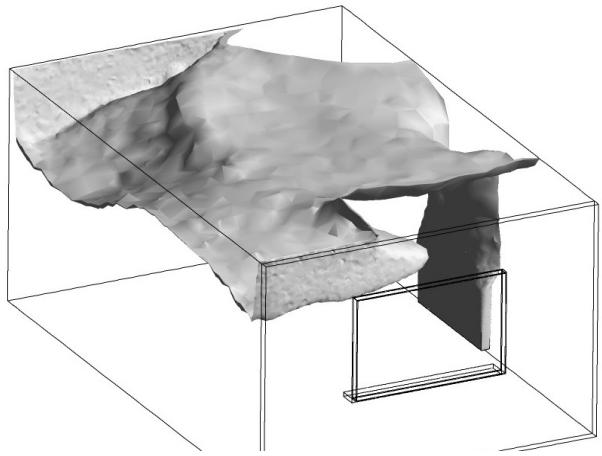

Fig. 8. Temperature isosurface of 20 ºC for model 3

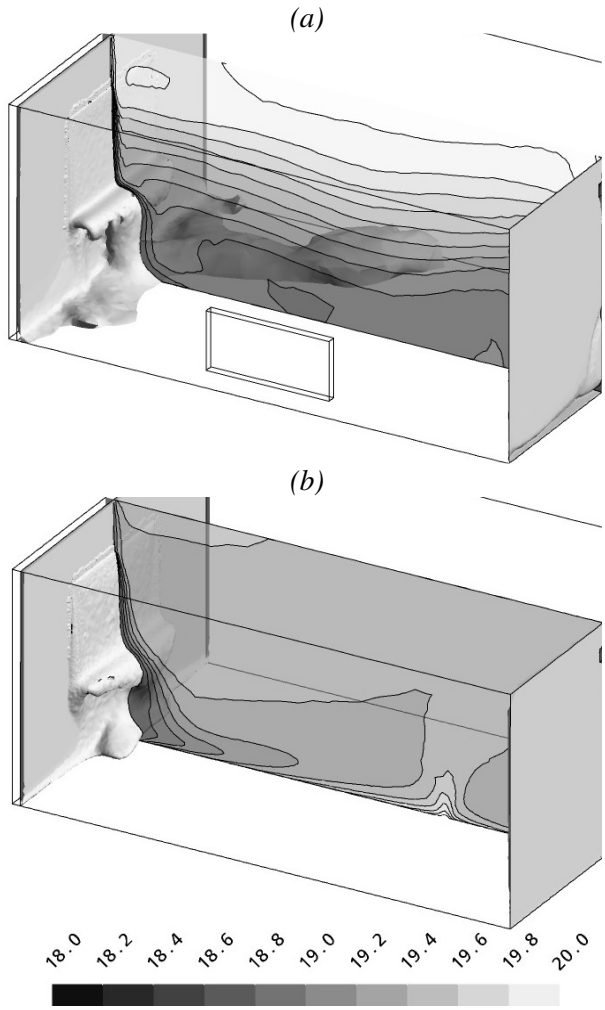

Fig. 9. Temperature contours from 18 to 20 ºC and temperature isosurface of 18 ºC for model 3 (a) and model 4 (b)

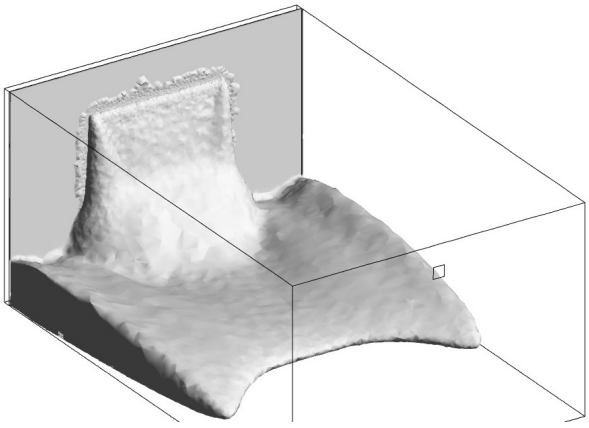

Fig. 10. Temperature isosurface of 14 ºC for model 4u

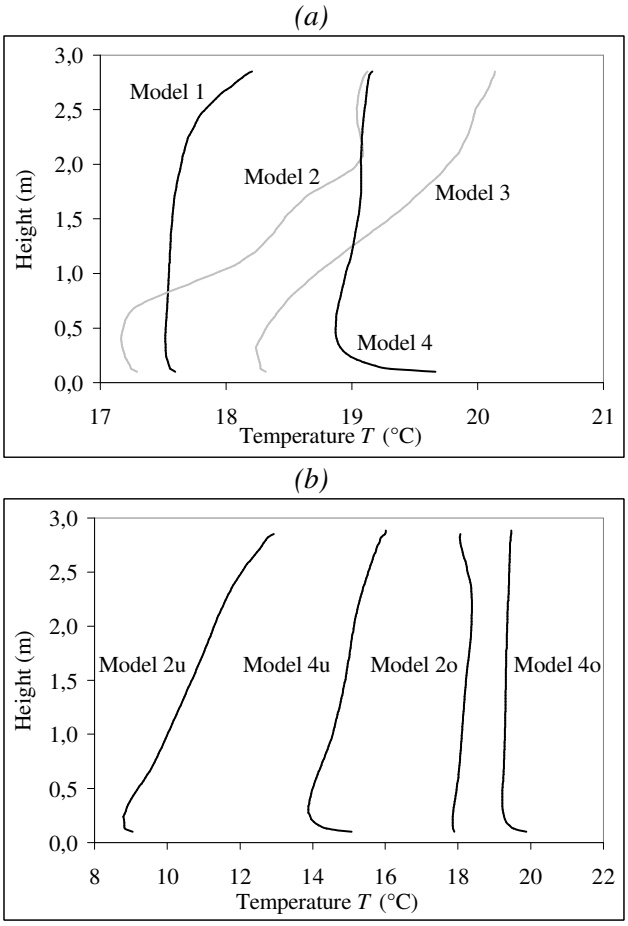

Fig. 11. Temperature profiles in the middle of the room for models 1-4 (a) and 2o, 2u, 4o, 4u (b)

Figure 11a comparatively shows the temperature profiles along a vertical line in the middle of the room for developed models with zero pressure difference. As it is evident from this visualisation and Table 1, the average temperatures in the room for models 3 and 4 are the same, but floor heating provides better comfort conditions due to insignificant fluctuations in the temperature in vertical direction.

Temperature profiles for the rooms with overpressure and underpressure are shown of Figure 11b. A tendency is clearly visible – for the models with underpressure absolute temperature values are lower and the air stratification is more considerable. At the same time difference in profiles for overpressure and underpressure conditions in the room with floor heating is less than for the room with convector heating.

From the viewpoint of the mean temperature and temperature fluctuations, floor heating is best-suited for human living, nonetheless the others models with zero pressure difference also meet the requirements of the conditions for thermal comfort [1]; only models with underpressure in the room do not satisfy the above mentioned conditions. In this way, complex analysis of the temperature field and the airflows as another parameter is needed.

According to the specification of the conditions for thermal comfort [1], the maximum airflow velocity in the heated room is limited to 10 cm/s, but it must be as small as possible in practice, except for the areas near openings and walls not used for human occupancy. As one can see from the results summarised in Table 1, the intensity of air flow is actually low (2…7 cm/s) throughout the room for all models, but the lowest velocities are observed in models 2 and 4.

Complex analysis of both thermal comfort conditions – temperature amplitude and airflow velocities (Fig. 7) shows that only one of these factors is at minimum in models 1, 2, 2o, and 4o; in models 2u, 3, and 4u – none of the parameters is at minimum; however, the best conditions for human living are observed in model 4. At the same time, this type of heating uses electric power and therefore it is related with greater expenses.

3.3 Risk of Condensation

A dew-point in living rooms can be reached near cold surfaces; particularly high risk of condensate appearance exists for the outside building structures having a high heat permeability. In this aspect, the most critical construction for simulated models is the window with heat transmittance U=2.5 W/(m^2·K). Such risk is increasing with difference between the characteristic temperature of the room and the temperature of the window surface.

In model 1, the characteristic room temperature exceeds 17 °C, while at the upper edge of the window the temperature falls down to 10 °C (Fig. 3a). As a result, the condensation on the window surface is highly probable; if the relative air

humidity in the room is 65 %, the condensation will begin at a temperature below 11 °C. Also in the rooms with underpressure (models 2u and 4u), the temperature near the outer wall is below 10 °C due to intensive cold exterior air inflow (Fig. 12), but here the dew-point can be reached at much lower temperature because of the low moisture content in outdoor air inflowing through the slots in a window.

Rather unexpected is the result that the risk of condensation on the window surface is practically absent in the case when the heater is placed by the wall opposite to the window (model 2). This is determined by the warm airflow along the ceilings in the direction to the outer wall and relatively immobile warm air masses in the upper part of this wall above the window (Fig. 3b). Since the heat insulation of the outer wall is chosen relatively good with heat transmittance U=0.35 W/(m^2·K) for all models, the probability of condensation is low nearby it. However, the risk of condensation could increase at the bottom part of the outer wall if its heat permeability is increasing.

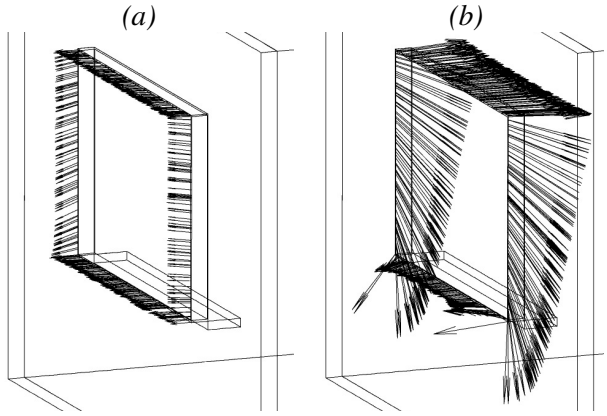

Fig. 12. Airflows through the crannies in the window-frame for models with overpressure (4o) and underpressure (4u)

4 Conclusion

3D numerical calculations of temperature and airflow distribution in a living room with an opening for air exchange show the essential influence of heater location, its type and pressure conditions on thermal comfort conditions in the room as well as on the heat transfer from the heater with constant surface temperature. Obtained temperature distributions help to forecast critical places near the boundary constructions where there is a high risk of condensation.

To summarise among all the considered models, the least advantageous from a viewpoint of thermal comfort conditions are the models with convector placed near the wall to the corridor and the side wall (due to great vertical temperature difference) and the models with underpressure in the room (with intensive cold air inflow), but the most advantageous – the model with the floor heating and without a pressure difference. In the last one, the temperature vertical difference is below 1 ºC and air velocities are only 3 cm/s, therefore the requirements of thermal comfort is satisfied the best of all. However, notable heat losses are observed here and since electricity is usually used for the floor heating, expenses for this type of heating are serious. Therefore, one must choose the most significant goal: to minimise the heating power or to improve the thermal comfort in the room.

Detailed analysis of modelling results for living rooms with different boundary constructions′ heat transfer coefficients, varying heater surface temperature, geometry configurations and corresponding comparison of heat balances as well as influence on thermal comfort conditions are analysed in publications [5, 6].

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