APPLICATIONS OF 3D MATHEMATICAL MODELS FOR IMPROVEMENT OF THERMAL COMFORT CONDITIONS IN LIVING ROOMS

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The paper deals with 3D mathematical models of a living room and the appropriate numerical calculations for characteristic thermal comfort conditions and distributions of temperature and averaged turbulent airflows using Ansys/CFX software. Many different models are developed and their results analysed depending on the geometrical properties (the placement of the heater and the windowsill) and ventilation conditions (pressure difference between opposite walls). The authors analyse the influence of these factors on the absolute temperature and its distribution as well as on the airflows – their velocities and directions in the room; therefore the main conditions ensuring thermal comfort for inhabitants are discussed. The thermal balance in a room composed of the heat flows through building structures and openings and its dependence on various external factors are also considered. The paper shows that it is possible to reduce heat consumption at the same time maintaining the conditions of thermal comfort in the room.

1. INTRODUCTION

Person’s comfort feeling in a living room is fundamental impressed by velocity of airflows, absolute temperature and amplitude of the vertical temperature gradient in the room [1,2]. The optimal arrangement of heaters, appropriate packing of window-frames and installation of a controllable ventilation system allows maintenance of thermal comfort in the living room with reduced heat consumption. Influence of the above mentioned factors is analysed in detail using the mathematical modelling approach. The proposed physical model of heat balance in a living room is considered for various physical conditions and geometry, which allows analysing the distributions of airflows and temperature. The mathematical model enables one to choose an optimal placement of building elements in order to decrease heat losses and improve conditions of thermal comfort.

2. METHODS AND MODELS

A room with different boundary conditions (convection, surface temperature, air openings) is modelled, which helps to understand the peculiarities of heat transfer process in the room as well as distribution of various characteristic quantities and their geometrical and physical dependences. In the 3D calculations multiple parameters are varied and their influence on the distributions of temperature and velocity fields characterising the conditions of thermal comfort is analysed. For numerical calculations the Ansys/CFX software is applied.
The calculations have been performed for the room shown in Fig. 1 filled with air. The window and the wall facing the exterior air are modelled using different heat transfer coefficients $U$; 2.5 W/(m²K) for the window and 0.35 W/(m²K) for the walls. On all external boundaries of the room the convection conditions are set with corresponding surface heat transfer coefficients [3]: for the boundary with exterior air – heat exchange coefficient $\alpha_{\text{out}} = 23.3$ W/(m²K) and for the boundaries with neighbouring rooms – heat exchange coefficient $\alpha_{\text{in}} = 8.1$ W/(m²K). It is conditionally assumed that the neighbouring spaces (upstairs, downstairs and side rooms) have a comfortable temperature of 20 °C, but that the end wall is contiguous with a corridor or a staircase where the temperature is lower (15 °C). The outdoor temperature is chosen corresponding to the winter conditions (−10 °C). The surface temperature of the heater is set constant (50 °C). On the surfaces of crannies in window-frame and ventilation system’s opening the boundary conditions are defined with constant pressure and corresponding temperatures of −10 °C and 15 °C. The pressure difference between opposite walls is set equal to 0 or ±1 Pa (see Table 1). For all surfaces, except openings, non-slip boundary conditions are assumed. The airflow in the room depends both on the convection created by the temperature difference and on the air exchange between openings in the building structures.

To describe the quasi-stationary 3D behaviour of temperature and averaged turbulent flows, traditional differential equations are employed [4]:

- the Reynolds averaged momentum equation;
- the continuity equation;
- the equations for specific turbulence energy $k$ and dissipation rate of this energy $\varepsilon$;
- the energy conservation equation.

The turbulent viscosity is calculated using the $k-\varepsilon$ turbulence model under traditional boundary conditions [5].

![Fig. 1. Layout of a modelled room and illustration of discretisation](image)

The discretisation was performed with tetrahedral elements of varying size; boundary layers are discretised with smaller prismatic elements. The characteristic size of finite elements is from 10 cm in the middle of the room to 0.3 mm in the vicinity of the heater and the openings in the walls. Therefore, the total number of elements depending on the geometry reaches the value of $6 \times 10^5$. The boundary conditions of the third type (convection from walls) and the low viscosity of air ($\mu_0 = 1.83 \times 10^{-5}$ kg/(m·s)) essentially worsen the convergence of the iteration process. The time required for calculations with a 3 GHz computer is about 3 days. The difference between the heat received from the heater and the heat losses from the outer surfaces and openings decreases below 10%.
3. RESULTS

Eight typical situations have been selected for modelling the temperature distribution and airflows in a dwelling house (see Table 1), which can be divided into four groups:

- Group A: along the window-frame a slot exists through which, depending on the pressure difference with the ventilation opening by the opposite wall, the cold air from outside can flow indoors or the warm air from inside – outdoors (variants 1–3);
- Group B: under the window there is there is no sill (variants 3, 4);
- Group C: the heater is placed under the window (which is traditional) or by the inner wall opposite to the window (variants 3, 5);
- Group D: the heater is placed by the inner wall opposite to the window and pressure difference is varied (variants 5, 6, 7).

<table>
<thead>
<tr>
<th>Modelling variant</th>
<th>Group of variants</th>
<th>Pressure difference between opposite walls ΔP (Pa)</th>
<th>Windosill Placement of the heater</th>
<th>Total heat amount received from the heater Q (W)</th>
<th>Average air velocity in the room v (m/s)</th>
<th>Average temperature in the room T (°C)</th>
<th>Vertical temperature difference in the middle of the room ΔT (°C)</th>
<th>Horizontal temperature difference in the middle of the room ΔT (°C)</th>
<th>Air exchange rate n (1/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 A</td>
<td></td>
<td>1 + W</td>
<td></td>
<td>253</td>
<td>0.06</td>
<td>13.7</td>
<td>4.1</td>
<td>9.2</td>
<td>1.5</td>
</tr>
<tr>
<td>2 A</td>
<td></td>
<td>−1 + W</td>
<td></td>
<td>167</td>
<td>0.06</td>
<td>18.5</td>
<td>0.7</td>
<td>2.8</td>
<td>1.5</td>
</tr>
<tr>
<td>3 A B C</td>
<td></td>
<td>0 + W</td>
<td></td>
<td>176</td>
<td>0.05</td>
<td>17.6</td>
<td>0.7</td>
<td>6.4</td>
<td>0.86</td>
</tr>
<tr>
<td>4 B</td>
<td></td>
<td>0 − W</td>
<td></td>
<td>199</td>
<td>0.04</td>
<td>19.6</td>
<td>1.4</td>
<td>2.3</td>
<td>0.21</td>
</tr>
<tr>
<td>5 C D</td>
<td></td>
<td>0 + O</td>
<td></td>
<td>145</td>
<td>0.02</td>
<td>18.3</td>
<td>2.0</td>
<td>4.6</td>
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</tr>
<tr>
<td>6 D</td>
<td></td>
<td>1 + O</td>
<td></td>
<td>209</td>
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<td></td>
<td>−1 + O</td>
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<td>145</td>
<td>0.04</td>
<td>18.2</td>
<td>0.5</td>
<td>4.3</td>
<td>1.4</td>
</tr>
</tbody>
</table>

W – near window, O – near opposite wall (to the corridor)

3.1 General results

Since the convective heat transfer from the heater is essentially dependent on the air flow intensity near its surface, it is obvious that, despite its constant temperature, the maximum heat will be taken off when along it a vigorous air motion occurs (variant 1); whereas at placing the heater by an inner wall (variants 5, 7) the released heat amount will be only 57% of the above mentioned because of slow airflows near the heater.

From the viewpoint of thermal comfort it is important that the temperature difference in the vertical and in the horizontal directions is as small as possible. In variants with underpressure in the room with cold air inflow through gaps in the window frame, air stratification with the vertical temperature difference more than 4 °C is observed (variants 1 and 6). An especially cold zone is formed near the floor by the window, since this zone is considerably cooled by the downstream cold air that enters along the window glass and
through slots in the window joints. Therefore for people it is uncomfortable to be in a zone closer than 0.5 m from the window, since there is an air temperature below 10 °C and there is a strongly felt flow of cold air (up to 35 cm/s), which in everyday life is known as “a draught blowing over one’s feet”.

The highest average temperature in the room is calculated in variant 4 (without windowsill) and the lowest – in variant 6 with a cold air inflow and the heater placed near the wall to the corridor (see Table 1 and Fig. 2). The heat amount received from the heater in variant 1 is maximum, but the temperature in the middle of the room falls down to 14 °C. The velocity of cold air flow near the window exceeds 50 cm/s, and the maximum intensity of this flow is retained high also in the middle of a room (20 cm/s). It is clear that to feel comfortably under such conditions is hardly possible.

Fig. 2. Total heat amount form the heater and characteristic average temperature for different modelling variants

Variant 4 could be considered as the most optimal of all from the viewpoint of thermal comfort conditions: the characteristic room temperature in this model reaches 19.6 °C, convective heat losses are low and the average value of airflow velocities in the room is below 4 cm/s. At the same time, horizontal and vertical temperature gradients are very slight (Fig. 3), but the heat amount taken from the heater is not the least one (Fig. 2). However one may conclude that this modelling variant possesses the optimum ratio of thermal comfort conditions and heat losses.

Fig. 3. Average velocity in the room and temperature differences in the middle of the room for different modelling variants
Especially high risk of condensate appearance exists for the outer building structures that have a high permeability of heat. In this aspect the most critical construction is the window ($U = 2.5\,\text{W/(m}^2\text{K})$). Such a risk is increasing with the difference between the characteristic temperature of the room and the temperature of the window surface: when the characteristic room temperature is quite high, at the upper edge of the window the temperature falls down up to 10 °C. As a result, the condensation on the window surface is highly probable; since, if the relative air humidity in the room is 60%, the condensation will begin at a temperature below 11 °C. It should however be noted that through the slots in window joints there is inflow of cold air whose absolute moisture content is lower. Therefore the condensation may take place in the bottom part of the window where the air from outside when flowing over this surface has already been mixed up with the more humid room air. This problem is essential for the room without window-sill (variant 4) and for the rooms with cold air inflow (variants 1 and 6) – see Figs. 4a and 4b.

![Fig. 4. Temperature isosurfaces of 17 °C for variant 1 (a) and of 8 °C variant 6 (b)](image)

Another significant condition for attaining comfort is air exchange in the room which is necessary for maintaining the content of oxygen inhaled by people. As the normal value characterizing air exchange intensity in the rooms without forced ventilation, the air exchange rate $n = 0.7\,\text{l/h}$ is accepted. Taking into account the air inflows and outflows through slots in the window-frame and through the ventilation opening it is obvious that in variants 1–3 and 6–7 the air exchange is sufficient (see Table 1). In turn, an air exchange above the normal would cause considerable heat energy losses (especially in variants 1–2 and 3–4), at the same time not making people feel better, since it decreases the temperature in the room and increases velocities of airflows. It is clearly shown that convective heat losses play significant role in the total heat balance: indeed, in modelling variants with highest values of average room’s temperature and the lowest heating energy (variants 4 and 5) the air exchange rates are the smallest (see Table 1).

To summarize, among all the considered modelling variants the least advantageous from the viewpoint of thermal comfort conditions are variants 1 and 6, and the most advantageous – variant 4 (see Table 1 and Figs. 2, 3).

Detailed analysis of 2D modelling results for living rooms with different heat transfer coefficients of boundary constructions obtained varying heater surface temperature as well as comparison of the heat balances is given in [6].
3.2. Analysis of the results for Group A

For the group of models with the heater placed near a wall to the outside and with a window-sill the greatest heat losses are observed in the case of underpressure in the room (variant 1), when there are cold airflows (temperature of –10 °C) through gaps in the window frame. In the model with overpressure in the room (variant 2), as well as for the variant without pressure difference between the window slot and the ventilation opening (variant 3) the heat needed for the heating is 30% lower (see Table 1). At the same time, the temperature distribution in height for the variant without pressure difference is very uneven. This is caused by active cold and hot air flows, which are partially separated by a windowsill and directed horizontally. When the air warmed by a heater is moving along its surface upward, it meets an obstacle – a windowsill, as a result of which the direction of a hot air stream is changed. But at the opposite wall of the room there exists downward inflow of cooler air through the ventilation opening.

Airflow and temperature distributions in the middle cross-section of the room for variants 1–3 are shown in Figs. 5–7. The corresponding temperature profiles for these variants can be seen in Fig. 8. For variant 1 a high risk of condensate exists near the inner surface of the window where the temperature reaches only 5 °C, which, in the case of high air humidity, means existence of dew point and moisture on the surface.

The air exchange rate for models of this group is quite high and, if there is pressure difference ΔP, its value does not depend on the direction of this difference (Table 1). In the case of ΔP=0 Pa the air infiltration is slowing down and the value of air exchange is 0.86 (1/h), which is normal for the rooms without forced ventilation.

\[ Fig. 5. \text{Airflows (a) and temperature contours form 10 to 20 \degree C (b) in the middle cross-section for variant 1} \]
Fig. 6. Airflows (a) and temperature contours form 10 to 20 °C (b) in the middle cross-section for variant 2

Fig. 7. Airflows (a) and temperature contours form 10 to 20 °C (b) in the middle cross-section for variant 3
3.3. Analysis of the results for group B

From the viewpoint of ensuring the thermal comfort the most advantageous situation in the room will be created when the outdoor pressure difference is zero (variant 3) and the heater is not covered by a windowsill (variant 4). The intensity of airflow is low (< 5 cm/s) practically all over the room (Fig. 9a). As one can see from the results in Table 1, the temperature in the room for variant 4 is the highest as compared with those in other variants, i.e. it is the closest to the temperature in the neighbouring rooms (20 °C) – see Fig. 9b. In this case the temperature difference in height in a larger part of the room is less than 1.5 °C (Fig. 10).

Fig. 8. Temperature profiles in the middle of the room for variants 1-3

Fig. 9. Airflows (a) and temperature contours form 10 to 20 °C (b) in the middle cross-section for variant 4

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In variant 4 (without a windowsill) the warm air flow is heating the inner surface of the window thus fully eliminating the risk of condensate appearance on this surface. In this variant the heat inflow from the neighbouring rooms where the temperature is conditionally assumed to be 20 °C is practically eliminated. In all other cases a significant heat inflow from contiguous rooms is observed.

Because of reduced air exchange in variant 4, in such rooms – especially if there are several inhabitants - an additional ventilation system might be required (or, otherwise, windows should often be opened). At the same time, convective heat losses in this case are insignificant (see Table 1).

3.4. Analysis of the results for group C

Somewhat unexpected has been the result that the risk of condensation on the window surface is practically absent in the case when the heater is placed by the wall opposite to the window (variant 5). This is determined by the warm air flow along ceilings in the direction to the outer wall and relatively immobile warm air masses in the upper part of this wall above the window (Fig. 11). In this case the risk of condensation could increase in the bottom part of the outer wall if its heat permeability is increasing. Since in all variants the heat insulation of the outer wall is relatively good (U=0.35 W/m²K), the probability of condensation near this place is low.

In this modelling variant air flows in the room are the slowest – only 2 cm/s (Fig.11a). Comparison of the results for the models without pressure difference and different heater locations (variants 3 and 5) shows that the highest average air temperature is observed in the case when the heater is placed near the wall to the corridor, but when it is placed near the window there will be the least vertical temperature difference (Fig. 12) and a considerable sufficient air exchange rate. Therefore choosing a proper place for the heater installation is very important – and not only for esthetical aspects but also for the reduction of heat losses and for the improvement of thermal comfort conditions in the room.

Similarly to variant 4, also in this modelling variant the heat losses by convection are low, therefore here also additional ventilation is needed because of deficient air exchange – see Table 1.
3.5. Analysis of the results for group D

The situation is more disadvantageous when in a room with the heater placed near the wall to the corridor pressure differences between the exterior air and the corridor exist (variants 6 and 7) – in this case the vector field and temperature distribution (Figs. 13–14) are very similar to variants 1 and 2, but the absolute values of air velocity, temperature and total amount differ (Table 1). However, the absolute temperature in variant 6 is only 10.6 °C owing to cold air inflow throughout the room along the floor (Fig. 4b).
Fig. 13. Airflows (a) and temperature contours form 10 to 20 °C (b) in the middle cross-section for variant 6

Fig. 14. Airflows (a) and temperature contours form 5 to 15 °C (b) in the middle cross-section for variant 7
Similarly to the case of a heater placed by the window (variants 1–3), for the heater arrangement near the inner wall to the corridor (variants 5–7) the thermal conditions in the room are influenced by pressure differences between gaps in the window frame and ventilation opening walls. Here we have the following situations:

- The cold inflowing air (−10 °C) through the gaps in the window frame (variant 6, Δp=1 Pa) creates a very low average temperature in the room (down to 10 °C). At the same time, the total heat amount from the heater is 50 % greater than in variants 5 and 7 (Δp=0 and -1 Pa). Besides, a considerable non-uniformity in the temperature distribution arises (e.g. more than 4 °C in the vertical direction – see Fig. 15), which makes this variant the least advantageous from the thermal comfort viewpoint.

- The cool air from the ventilation system (15 °C) is mixed with the hot airflow from a heater near the room’s ceiling (variant 7, Δp=-1 Pa). As a result, the flow of a colder air from the ventilation opening is limited and a significant airflow exists only near the ceiling. Therefore, the vertical temperature distribution in the room is uniform (Fig. 15) expect near the outer wall and in the heater’s zone; here the average characteristic temperature in the room practically does not change (18.2 °C) in comparison with variant 5 without pressure difference, at the same time the air exchange rate in variant 7 is 4 times greater than in variant 5 and is the same as in variant 6 (Table 1).

4. CONCLUSIONS

The 3D numerical calculations of temperature and airflow distribution in a living room with openings for air exchange have shown a strong influence of windowsill and heater arrangement on the thermal comfort conditions in a room and on the heat transfer from a heater with constant surface temperature. It is established that the most advantageous situation is observed in variant 4 with no windowsill and where pressure difference between the exterior air and the ventilation opening is absent is exist. In variants 1 and 6, at the overpressure of 1 Pa on the outer wall opening low temperature zones are developing in the room and there exists an expressed temperature stratification; in this case relatively high air velocities are observed, which means an increased heat transfer from the heater. The temperature distributions obtained can help to forecast critical places near boundary constructions where high risk of condensation exists.

The model of a separated room shows the influence of various factors on the resulting distributions of thermo-physical parameters in the room, which is directly
associated with the conditions of thermal comfort. At the same time, the model shows possibilities to reduce heat consumption, which would mean decrease in the energy production and in the related pollutions.

REFERENCES


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