LATVIAN JOURNAL OF PHYSICS AND TECHNICAL SCIENCES
2004, N 4

NUMERICAL MODELLING OF HYDRAULIC RESISTANCE IN THE PIPES OF DIFFERENT GEOMETRIC SHAPES

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Hydraulic resistance of pipelines, being strongly affected by their cross-section shape in the longitudinal direction and curvature, is an essential factor in technological processes connected with transportation of liquids and gases. To determine this resistance in the cases of pipelines with complicated configuration it is necessary to carry out numerical modelling of the flow. The accuracy of the numerical calculations was estimated applying ANSYS/Flotran software. The hydraulic resistance of a pipeline was analysed depending on the curvature, cross-section shape and its variation in the longitudinal direction as well as on the viscosity of fluid. As could be expected, the resistance of low-viscosity flows (possessing high Reynolds' numbers) was considerably increasing with pipeline bending and with longitudinally varying cross-section. The hydraulic resistance in non-deformed pipelines was found to correspond to the values determined analytically.

1. INTRODUCTION

In various technological equipment the running of the processes and their efficiency is often determined by viscous fluid flowing in the pipes of different type – e.g. pipelines for water supply and heating, for transport of oil products, pipes dealt with in chemical and food industry, etc.. A specific class of flows where hydraulic resistance influences considerably the quality of product and the duration of technological cycle are those where composite materials are impregnated with several reinforcing layers of polymer resin. If the reinforced fibre beams in separate layers are oriented perpendicularly to each other, in the inter-beam gaps a periodic canal-like structures in the flow. Such flows, following the curvature of the canals and the strongly varying cross-sections, are spatial, therefore for detailed description three-dimensional mathematical modelling with applying Navier-Stokes equations is required. Since the corresponding calculations are cumbersome, it is worthwhile to analyse typical periodical structures of the flow, approximating them as flows in the pipes with various cross-sections and curvature as well as with longitudinally varying cross-section areas. Such a simplified approach when describing complicated flows facilitates the analysis of significant factors influencing these flows and is easier to apply in calculations that are to be performed for optimisation of the processes. The paper considers, first of all, laminar flows in the pipes with constant cross-section, in order to estimate the accuracy of numerical calculations and to demonstrate the influence of various factors on the flow. After that, the curvature effect in the flow is analysed depending on different Reynolds numbers for laminar flows. Finally, calculations of hydraulic resistance were performed for the pipelines with variable cross-section.

2. ANALYTICAL SOLUTIONS

Consider the flow of fluid in a straight pipe with constant cross-section and its shape shown in Fig. 1. The pipe's hydraulic resistance coefficient C (1/m^2) in general is defined as follows [1]:

$$C = \frac{S\,\Delta P}{\mu\,L\,Q}, \tag{1}$$

where S (m^2) is the pipe's cross-section area, ΔP (Pa) is the pressure difference between its both ends, μ (Pa·s) is the fluid viscosity, L (m) is the canal length and Q (m^3/s) is the throughflow (in the literature hydraulic permeability K=1/C (m^2) can also be found, which is the inverse of C).

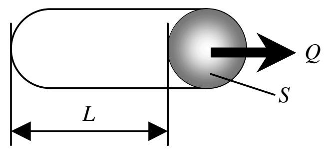

Fig. 1. Pipe elements used for calculation of hydraulic resistance

Consider a laminar flow of the fluid with a given viscosity in a straight pipe, whose hydraulic resistance depends only upon the cross-section geometry (shape). This flow is determined by the dependence of fluid velocity distribution and therefore of the friction force on the pipe walls on the shape of the cross-section. At a fixed cross-section area S the least hydraulic resistance will be in a pipe of circular shape [2]:

$$C = \frac{8}{R^2}, \tag{2}$$

where R (m) is the circle radius, since the corresponding throughflow is defined by the formula:

$$Q = \frac{\pi R^4 \Delta P}{8\mu L}. \tag{3}$$

Considering the pipe of elliptic cross-section with semi-axes a and b in a similar way, the expressions for throughflow and hydraulic resistance coefficient will be as follows [2]:

$$Q = \frac{\pi a^3 b^3 \Delta P}{4\mu L(a^2 + b^2)} \text{ and } C = \frac{4\left(a^2 + b^2\right)}{a^2 b^2}. \tag{4), (5}$$

In the case of rectangular cross-sections, the flow and resistance calculations become more complicated, and for a rectangle with the sides $2h$ and $2\chi h$, where χ is the ratio of side lengths (χ>1) [2], the throughflow will be:

$$Q = \frac{\Delta P}{4\mu L}\chi h^4 f(\chi), \tag{6}$$

where $f(\chi) = \frac{16}{3} - \frac{1024}{\pi^5 \chi}\left(\text{th}\frac{\pi\chi}{2} + \frac{1}{9}\text{th}\frac{3\pi\chi}{2} + \ldots \right)$

and the hydraulic resistance coefficient:

$$C = \frac{16}{h^2}\frac{1}{f(\chi)} \ . \tag{7}$$

In an approximate calculation it is possible to restrict ourselves to two terms in the $f(\chi)$ expression, and with the ratio of cross-section sides increasing the changes in the resistance coefficient become insignificant:

χ	1	2	3	5	10	100	∞
$f(\chi)$	2.253	3.664	4.203	4.665	5.000	5.299	5.333

Thus, applying expressions (2), (5) and (7), it is possible to calculate analytically the hydraulic resistance coefficient for the pipes of different shape and simple cross-section. For the pipes with an arbitrary cross-section, especially for those with longitudinally varying cross-section, such simple analytic calculations become impossible, and here one of possibilities is to invoke numerical modelling of flows. The analytical formulas specified above can be used to estimate the accuracy of numeric calculations.

3. NUMERICAL CALCULATIONS

In our numerical calculations a 3D model of viscous incompressible laminar fluid and a package of programs ANSYS/Flotran have been employed [3], where the modelled pipe is discretised applying the finite-element-based method. In the case of straight pipes only the longitudinally directed velocity component $v_{\|}$ differs from zero, while in the case of a pipe's cross-section varying in the longitudinal direction the velocity components perpendicular to the pipe axis will also differ from zero, that is, $v_{\perp} \neq 0$.

Applying thus obtained velocity distributions at a fixed pressure difference between the pipe's ends one may calculate throughflow Q. Since this parameter for incompressible fluid, according to the substance conservation law, in all cross-sections is equal, in the throughflow calculation one can use the cross-section area (S) and the relevant velocities ($v_{\|}$) perpendicular to the cross-section at discrete points of an arbitrary cut, e.g. at the pipe's inlet:

$$Q = \int_{S_i} \vec{v}_i d\vec{S}_i = Q_{inlet} = \int_{inlet} v_{\|} dS \ . \tag{8}$$

Velocities at discrete points are calculated numerically, and, if the corresponding finite elements' areas ΔS are approximately equal, the throughflow at the inlet (and therefore also all over the whole pipe) can be approximately calculated by summing up the velocities in all n cross-section elements, i.e. applying the rectangle formula

$$Q_{numerical,1} = \int_{inlet} v_{\|} dS = \sum v_{\|}\Delta S = \Delta S \sum v_{\|} = \frac{S}{n}\sum v_{\|} \ . \tag{9}$$

Then the following numerical formula approximated for the hydraulic resistance coefficient (1) is conveniently applicable to the case of triangular elements:

$$C = \frac{n \Delta P}{\mu L} \frac{1}{\sum v_{\parallel}} . \tag{10}$$

If a pipe's cross-section at the inlet is a rectangle and there rectangular elements are used (whose division is regular in both dimensions), then the more accurate trapezium formula for numerical integration [4] in the throughflow calculation is convenient to use, replacing the throughflow Q in expression (1) for hydraulic resistance by the integral sum $Q_{numerical,2}$:

$$Q_{numerical,2} = h \cdot g \left\{ \left[\frac{v_{\perp}(x_0, y_0)}{4} + \sum_{j=1}^{M-1} \frac{v_{\perp}(x_0, y_j)}{2} + \frac{v_{\perp}(x_0, y_M)}{4} \right] + \right.$$

$$+ \left[\frac{v_{\perp}(x_N, y_0)}{4} + \sum_{j=1}^{M-1} \frac{v_{\perp}(x_N, y_j)}{2} + \frac{v_{\perp}(x_N, y_M)}{4} \right] +$$

$$\left. + \sum_{i=1}^{N-1} \left[\frac{v_{\perp}(x_i, y_0)}{2} + \sum_{j=1}^{M-1} v_{\perp}(x_i, y_j) + \frac{v_{\perp}(x_i, y_M)}{2} \right] \right\} , \tag{11}$$

where h and g (see Fig. 2) represent the sizes of the element's sides.

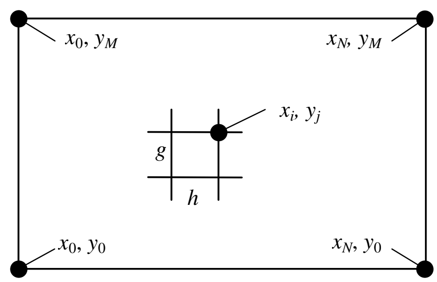

Fig. 2. Notations used in the throughflow approximate calculation

Velocities at the nodes with minimum or maximum values of coordinates x or y on the pipe walls, owing to the so-called non-slip conditions, are equal to zero, therefore expression (11) becomes simpler:

$$Q_{numerical,2} = h \cdot g \sum_{i=1}^{N-1} \left(\sum_{j=1}^{M-1} v_{\perp}(x_i, y_j) \right) . \tag{12}$$

Applying formulas (9) or (12), depending on to the finite element form used in the calculations, approximated values for flow throughflow in the pipes of different geometrical shape could be obtained. Comparing the numerical results for the hydraulic resistance with the analytical ones, first of all one can verify the accuracy of numerical modelling for straight pipes with a simple cross-section.

4. PIPE CROSS-SECTION WITH VARYING SHAPE

To compare the numerical calculation results with theoretical ones, three different shapes for pipe cross-sections of equal area were chosen – circular, square, and rectangular with the side ratio $\chi = 9$ (see Fig. 3). Depending on the shape of the cross-section chosen, the finite elements of two types are used – triangular for circular cross-sections and square elements for square and rectangular cross-sections. The geometric parameters and physical characteristics of fluids used in the model are presented in Table 1. Since the hydraulic resistance for the modelled pipes is independent of viscosity, the fluid was chosen with a relatively high dynamic viscosity – $\mu = 1$ Pa·s. This was made to reduce the calculational time, which increases with viscosity decreasing.

The accuracy of numerical results essentially depends upon the throughflow calculation method; thus for rectangles numerical integration formula (11) is more precise, which is not applicable for a circle because of the shape of its finite elements. In this case the calculation is performed applying the approximate formula (9).

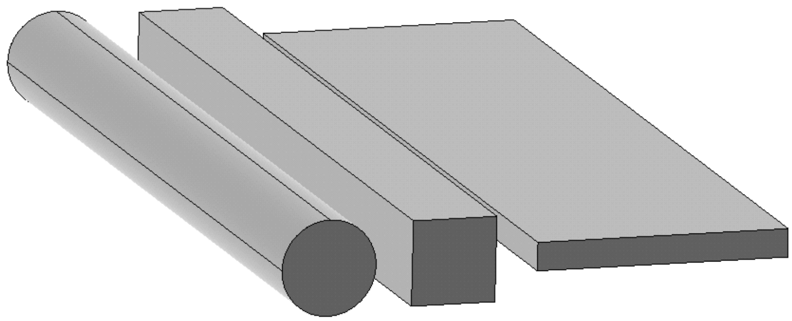

Fig. 3. Straight pipes with different cross-section shapes

Table 1

Geometric and physical parameters of the model

Item	Designation	Value	Measuring unit
Cross-section area for each pipe	S	0.01	m^2
Radius (for circle)	R	0.0564	m
Side (for square)	d	0.1	m
Shortest side (for parallelogram)	a	0.0333	m
Longest side (for parallelogram)	b	0.3	m
Pipe length	L	1	m
Pressure difference	ΔP	10	Pa
Density	ρ	1000	kg/m^3
Dynamic viscosity	μ	1	Pa·s

Comparison of calculation results with the theoretical ones is given in Table 2. As is seen, the agreement is rather good and the difference between the numerical calculation results and the corresponding analytical solutions does not exceed 5%, therefore, it is possible to assume that the numerical expressions describe the physical processes sufficiently well, and therefore they can be used in the calculations of differently shaped pipes where analytical calculations are impossible, e.g., for bent pipes and for those with a longitudinally varying cross-section. It should be noted that applying square elements it is possible to improve the accuracy of calculation by increasing their number.

Table 2

Analytical and numerical results for the pipes of differently shaped cross-section

Cross-section shape	Type of element	Number of elements in longitudinal direction (n)	Characteristic area of elements (m^2)	$C_{analytical}$ ($1/m^2$)	$C_{numerical,1}$ ($1/m^2$) formula (9)	$C_{numerical,2}$ ($1/m^2$) formula (12)
Circle	∇	13000	$7.6 \cdot 10^{-7}$	2515	2560	not used
Square	□	6250	$1.6 \cdot 10^{-6}$	2841	2880	2998
Rectangle	□	13000	$7.7 \cdot 10^{-7}$	11664	11654	12097

5. ANALYSIS OF THE PIPE CURVATURE AND VISCOSITY EFFECTS

We will consider pipes with equal square cross-sections (0.1 m×0.1 m) and of equal length but of different curvature – beginning with a straight pipe and ending with a pipe where the flow direction changes by 180° (Fig. 4). The curvature of the seven pipes considered is respectively: 0, π/4, π/2, π/1.75, π/1.5, π/1.25 and π (180°). In these pipes the flow of fluids possessing different viscosities is modelled, which allows the dependence of hydraulic resistance to be determined not only on their curvature but also on the velocity of the flow or Reynolds number (*Re*) [2]:

$$Re = \frac{v \, \rho \, d}{\mu}, \tag{13}$$

where v (m/s) is the average flow velocity in the pipe cross-section (being calculated from the velocity distribution at discrete nodes) and d (m) is a characteristic length in the cross-section (in this case it is assumed to be equal to the half-length of the square side).

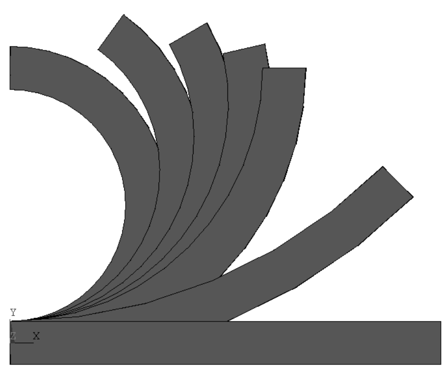

Fig. 4. Pipes of different curvature

In the calculations, the following series of dynamic viscosity values is used: μ = 1, 0.1, 0.05, 0.035, 0.02, 0.015 (Pa·s). The other data correspond to Table 1. For numerical integration of the fluid flow the trapezium formula (12) was applied, since as shown above it provides higher accuracy of square-element discretisation of rectangular cross-sections.

The results of numerical calculations depending on the fluid viscosity (or Reynolds number) and the pipe curvature are generalized in Table 3.

Table 3

Results of hydraulic resistance calculations for different viscosities and curvature

Curvature k (1/m)	Viscosity μ (Pa·s)						
	1	0.1	0.07	0.05	0.35	0.02	0.015
	Hydraulic resistance C (1/m^2)						
π	2859	3034	3309	3757	4338	5398	5741
π/1.25	2859	2978	3191	3570	4071	4946	5237
π/1.5	2860	2945	3115	3444	3891	4631	4802
π/1.75	2860	2924	3063	3353	3759	4405	4595
π/2	2859	2909	3025	3282	3653	4224	4409
π/4	2859	2871	2912	3029	3246	3564	3665
0	2860	2853	2856	2856	2856	2856	2856

At small Reynolds numbers or in the case of a slow flow the hydraulic resistance coefficient does not change with pipe curvature increasing, i.e. it does not depend on this parameter. At the same time, for relatively high-speed (but still laminar) flows at μ = 0.015 (Pa·s) the resistance coefficient in a pipe with a curvature of 180° (π) is twice as much as for straight pipes of the same cross-section. This is connected with an increased role of the convective transfer of the impulse in comparison with viscous effects that are more expressed as the velocity increases, which is proved by the velocity profile that at small viscosities (large *Re* numbers) is not parabolic any more and becomes asymmetric with respect to the central line of a pipe (see Fig. 5) – the velocity maximum is shifting to the wall side that is farther from the curvature centre. For laminar flows in a straight pipe with a constant cross-section the equality $(\vec{v}\cdot\nabla)\vec{v}=0$ holds irrespective of the flow velocity, hence the corresponding hydraulic resistance is independent of viscosity. Since in the calculations the laminar flow model is used, the flows with low viscosity are not considered for with velocity increasing they become unstable and turbulent.

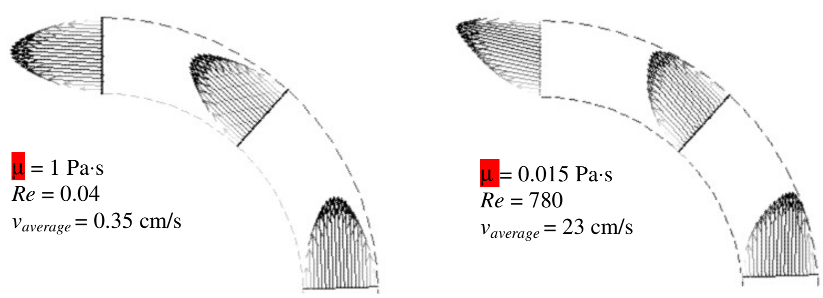

Fig. 5. Velocity distribution in the pipe cross-sections at different *Re* numbers

Figure 6 shows dependence of hydraulic resistance coefficient on the curvature. As is seen, the increase in this resistance can be expressed by the linear function: $C=a\cdot k+C_0$, where coefficient C_0 is the resistance of a straight pipe determined only by its cross-section shape and a is a function of the viscosity or *Re*.

Hydraulic resistance dependence on the viscosity is shown in Fig. 7. It is obvious that at high curvature of a pipe ($k = \pi$) the resistance rapidly falls from large values at small viscosity (large *Re* values) down to the C_0 value, which is the hydraulic resistance of a straight pipe. As shown above, in the pipes with high curvature the dependence on the viscosity is clearly expressed; while for pipes with a smaller curvature this is less expressed, therefore hydraulic resistance coefficient for a straight pipe does not depend on the viscosity (or Reynolds number).

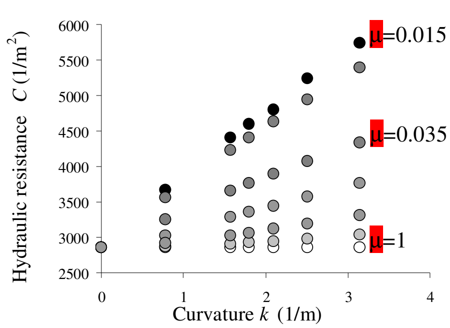

Fig. 6. Hydraulic resistance dependence on the curvature at different viscosities

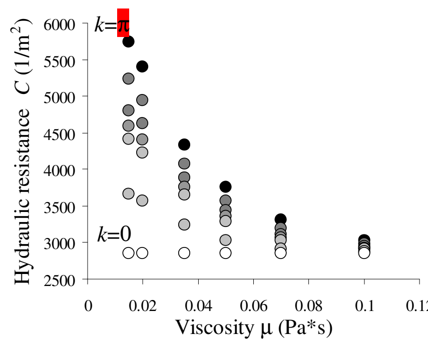

Fig. 7. Hydraulic resistance dependence on viscosity at different curvatures

Figure 8 shows the flow velocity or Reynolds number in dependence on the pipe curvature at different fluid viscosities. It is readily seen that in the case with a large viscosity (μ =1 Pa·s) the flow velocity does not depend on the pipe curvature and is low; at the same time, as viscosity decreases (μ = 0.015 Pa·s) the fluid velocity is growing and the dependence of Reynolds number on the curvature also becomes more expressed – the *Re* value is reduced by half within the interval from 0 to π. The straighter is a pipe, the higher velocity the flow achieves within, which determines the growth in Reynolds number.

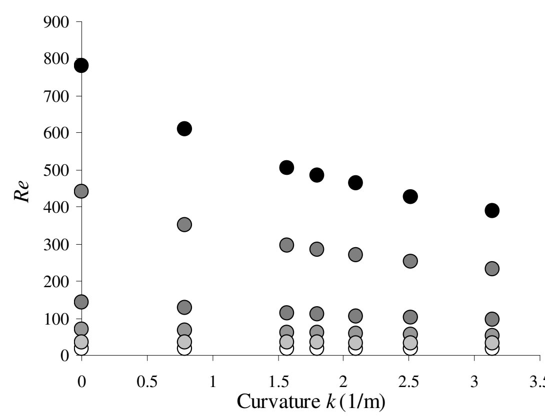

Fig. 8. Reynolds number dependence on the pipe curvature at different viscosities

6. LONGITUDINAL VARIATIONS IN RECTANGULAR CROSS-SECTIONS AND VISCOSITY EFFECTS

As the next pipeline model we will consider straight pipes with rectangular cross-section that is periodically varying along the pipe's length. Two types of such a periodical structure variation are modelled, and, correspondingly, two models are chosen: (A) – the pipes with cross-section variations only in one dimension (Fig. 9*A*) and (B) – the pipes where such variations occur in two dimensions (Fig. 9*B*). As characteristic parameter for the variations the ratio of the smallest and the largest sides of cross-section, a_{min}/a_{max} , is chosen.

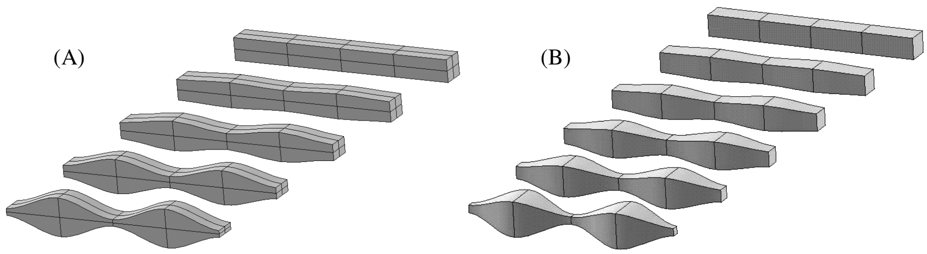

Fig. 9. Straight pipeline models with variable rectangular cross-section in one (*A*) and two (*B*) dimensions

In hydraulic resistance calculation we will apply the fluid flow calculated by formula (12), since because of the cross-section shape the discretisation is made with the rectangular elements and their distribution is regular. As a whole, the flows were modelled in both the models (A and B) applying the following fluid viscosities: μ = 1, μ = 0.3, μ = 0.1 and μ = 0.05 (Pa·s). Modelling at lower viscosities and the number of cross-section discretisation nodes used (about 2500) was impossible owing to the growth in the flow velocity, since the laminar model calculations were diverging.

Table 4

Hydraulic resistance at different viscosities and pipe cross-section variation amplitudes in A and B models

Model	Cross-section side ratio a_{min}/a_{max}	Viscosity μ (Pa·s)			
		1	0.3	0.1	0.05
		Hydraulic resistance C (1/m^2)			
A	1	2859	2850	2851	2867
	0.818	2893	2879	2797	2646
	0.538	3351	3317	3203	2845
	0.333	4817	4761	4536	4337
	0.176	10596	10510	9946	9746
B	1	2885	2850	2856	2856
	0.852	–	–	2951	2782
	0.695	–	–	3505	3392
	0.686	3532	3533	–	–
	0.550	–	–	4905	5290
	0.469	6492	6492	–	–
	0.399	–	–	9153	11026
	0.295	19804	19774	–	–
	0.212	–	–	43994	50409
	0.143	163152	162592	–	–

The results of numerical calculations for both models depending on the velocity of the fluid used and on the cross-section variation amplitude (ratio a_{min}/a_{max}) are presented in Table 4. For calculation of the hydraulic resistance C an average pipe cross-section, $\bar{S} = \frac{1}{2}(S_{min} + S_{max})$, was used. In both the models for a straight pipe with constant cross-section the hydraulic resistance at any viscosity approximately coincides with the value of 2841 ($1/m^2$) determined analytically by relationship (7). In turn, in the pipes with variable cross-section the velocity components arise in the perpendicular cross-section plane ($v_\perp$), and the hydraulic resistance grows at increasing flow velocity. In this case, the higher cross-section variation amplitude, the more pronounced resistance dependence is on the fluid velocity (or Reynolds number).

Qualitative calculation results for both the models are similar; therefore, we can present graphically the data for the more complicated configuration – that of model B. It should be noted that calculations of model B were made with different pipe side ratios for viscosities $\mu = 1/0.3$ and $\mu = 0.1/0.05$ (see Table 4).

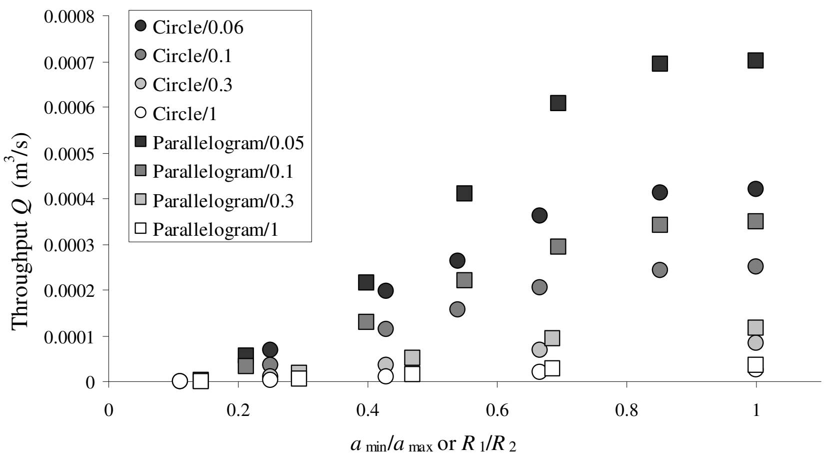

Fig. 10. Throughflow dependence on the cross-section variation amplitude for fluids of different viscosities for rectangular and circular cross-sections

Figure 10 shows dependence of throughflow Q on the variations in pipeline cross-section amplitude (designated with squares). As is seen, at the size ratio $0.8<a_{min}/a_{max}<1$ the flow intensity and hydraulic resistance change only slightly; at $a_{min}/a_{max}<0.7$ the flow intensity starts decreasing rapidly ($\mu = 0.05$ in the figure) while the hydraulic resistance grows very fast (see Fig. 11, where C is given in a logarithmic scale). Such growth is occurs mainly owing to rapid fall in the pressure (Fig. 12) required to ensure the fluid flow through the narrowest cross-section, since in the stationary case $v_{S_{max}} \cdot S_{max} = v_{S_{min}} \cdot S_{min}$, thus the maximum flow velocities in the cross-section with the minimum area (Fig.13) exceed many times the velocities in the cross-section with the maximum area. At high variation amplitudes in the cross-section area and low viscosity, this additional decrease in the throughflow is created by its more expressed spatial behaviour (Fig. 14*a,b*) and separation from the walls, that is, formation of "parasitic" vortices with a counter-stream appearing (Fig. 14*b*).

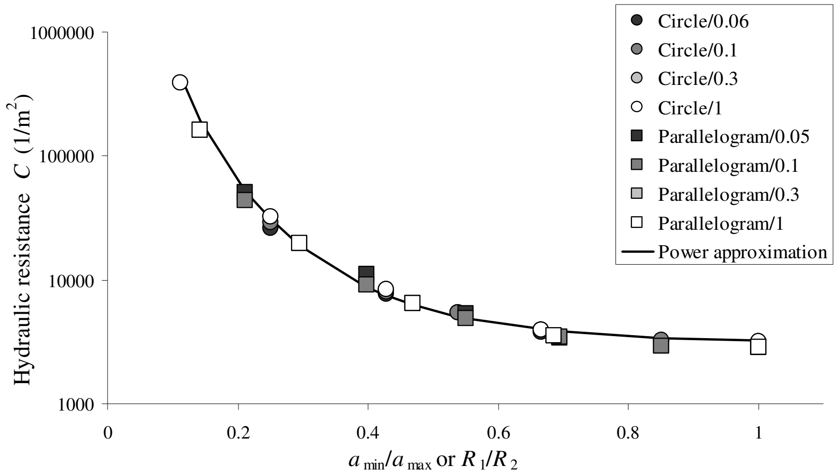

Fig. 11. Hydraulic resistance dependence on the cross-section variation amplitude for fluids of viscosities for rectangular and circular cross-sections (approximated with a power function)

The said above regarding the pressure fall in the minimum cross-section area is confirmed by the dependence of the hydraulic resistance C on $a_{\min}/a_{\max}$ in Fig._11. One can see there that differences in this resistance for fluids of low and high viscosity are insignificant since the resistance coefficient's dependence on the fluid viscosities is much weaker (by two orders of magnitude) compared with the dependence on the pipe cross-section variations (for approximation of this curve with an analytical function see the next chapter).

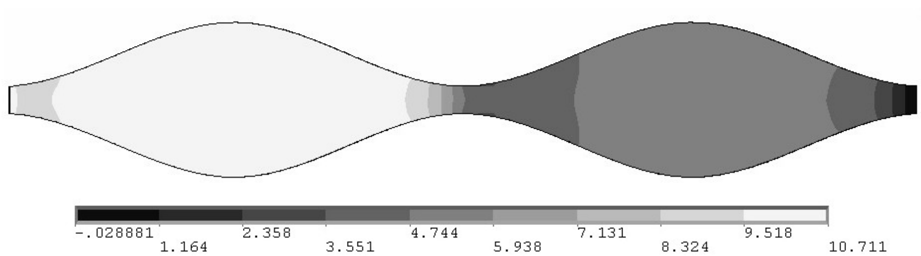

Fig. 12. Fluid flow pressure distribution in the medium sections

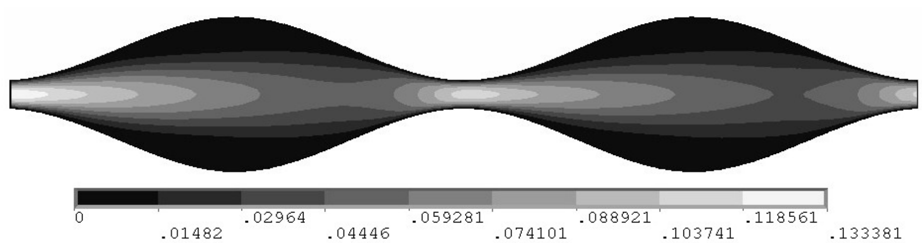

Fig. 13. Fluid total velocity distribution in the medium sections

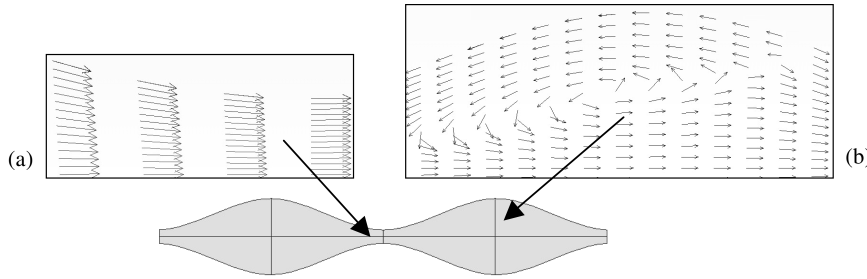

Fig. 14. Qualitative behaviour of the vector field of fluid velocities in the medium section in the widest (*a*) and narrowest (*b*) place of a pipe

7. CIRCULAR CROSS-SECTION VARIATIONS IN THE LONGITUDINAL DIRECTION AND VISCOSITY EFFECTS

Consider a pipe with circular cross-section whose area varies in the longitudinal direction. To determine the dependence of the hydraulic resistance of such a pipe both on the fluid viscosity and on the varying cross-section radius, we will consider a number of pipes with different maximum-to-minimum ratio of cross-section radii. In view of the pipe's shape, for discretisation a tetrahedron finite-elements' grid was applied (Fig. 15). For numerical integration of throughflow Q, and calculation of hydraulic resistance C, we will use approximate formulae (9) and (10), respectively. For calculating C as the cross-section area we will use the average value $\overline{S}$.

We will characterise various geometric shapes of pipes with the radius ratios for the widest and the narrowest places of a pipe thus obtaining the range for $R_{\min}/R_{\max}$ from 0.1 to 1. Here $R_{\min}/R_{\max}$=1 corresponds to a non-deformed pipe whose hydraulic resistance can be defined by analytical formula (2).

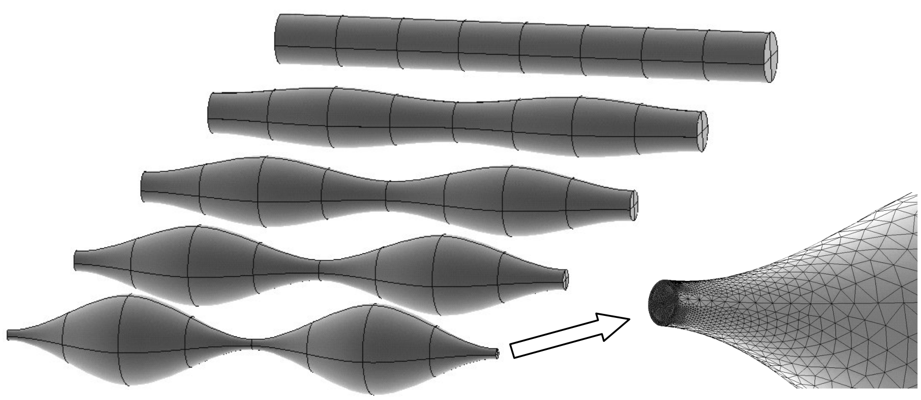

Fig. 15. Circular pipes longitudinally variable in the cross-section and their finite-elements' discretisation

Although the number of elements in the cross-section is relatively small and does not exceed 850, the results of calculations (see Table 5) can be considered sufficiently precise, as the difference between the value of C=3200 1/m^2 determined analytically by formula (2) and the numerical results for a straight pipe ($R_{\min}/R_{\max}$=1) is not greater than 3%. To control the results in each calculation variant the total throughflow was defined at the maximum and minimum cross-sections of a pipe – the difference did not exceed a few percent, which evidences that the model describing the process and its results are true. It should be noted that the calculations at the viscosities smaller than 0.06 (Pa·s) for the given discretisation were not possible any longer in view of numerical calculation divergence – the maximum velocity at the narrowest places reached 12 cm/s and the Reynolds number – 1700.

Table 5

Hydraulic resistance at various viscosities and pipe curvature

R_{min}/R_{max} ratio	Viscosity μ (Pa·s)							
	1	0.3	0.1	0.06	1	0.3	0.1	0.06
	Throughflow $Q \times 10^{-5}$ (m^3/s)				Hydraulic resistance C (1/m^2)			
1	2.5	8.3	25.2	41.9	3155	3172	3118	3121
0.85	–	–	24.3	41.3	–	–	3253	3189
0.67	2.1	6.8	20.5	36.1	3978	3979	3971	3770
0.54	–	–	15.8	26.4	–	–	5414	5412
0.43	1.1	3.6	11.3	19.8	8426	8417	8017	7661
0.25	0.3	1.1	3.7	6.8	32212	32105	29103	26182
0.11	0.03	0.1	–	–	388372	385548	–	–

Qualitatively, the dependence of throughflow and hydraulic resistance on the ratio of the pipe radii for fluids with various viscosities is quite similar to the results obtained for pipes with a rectangular cross-section area, that is why Figs. 10 and 11 show the results of both the models. As is seen, the values of hydraulic resistance (in the case of equal cross-section areas' proportion) for pipes of circular cross-section are less than for rectangular ones, which could be expected taking into account the analytical hydraulic resistance difference for straight pipes with circular and rectangular cross-sections (see Table 2). The dependence of hydraulic resistance on the cross-section characteristic ratio R_{min}/R_{max} is expressed very clearly, and therefore it is represented in a logarithmic scale. The growth in the hydraulic resistance is the most rapid as R_{min}/R_{max} ratio decreases below 0.5. One of the best approximations of this dependence (both for the cases of rectangular and circular pipes' cross-sections) is a power function $a + b \cdot x^c$ with the coefficients $a \approx 3000$, $b \approx 250$, and $c \approx -3.4$.

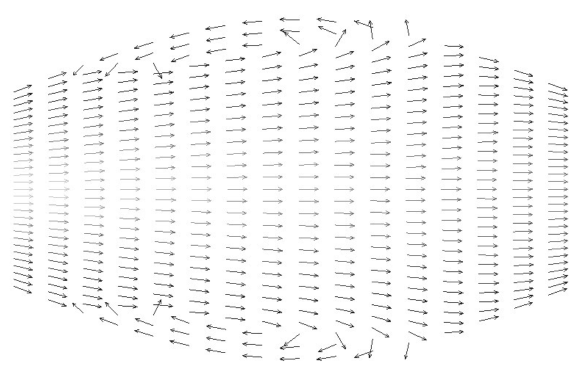

Fig. 16. Velocity vector-field's qualitative behaviour in the medium pipe section in its widest place

Similar to the case of a rectangular pipe, here the rapid growth in resistance is determined by high pressure fall at the narrowest place of the pipe (R_{min}). Also, in circular pipelines with a variable cross-section the flow separation from walls and a counter-stream arising owing to low fluid viscosity in the widest part of a pipe (Fig. 16) affects only slightly the hydraulic resistance growth, therefore its dependence on the viscosity is also insignificant.

8. CONCLUSION

The results achieved can be considered sufficiently well describing the processes, since the numerical calculations of hydraulic resistance in straight pipes with variously shaped cross-sections have shown good agreement with the analytical solutions, with the errors not exceeding 5%. In turn, the comparative analysis made for a fluid flowing in one pipe with different cross-sections shows the scatter to be only about 1%, which proves the numerical calculations to be converging.

For the model describing pipes of different curvature and square-shaped cross-section the authors have established the dependence of the linear hydraulic resistance on the curvature, i.e., with the pipe's curvature increasing its hydraulic resistance grows linearly. In pipes with a variable cross-section in the longitudinal direction the hydraulic resistance coefficient strongly depends on the cross-section variation amplitude, with its growth especially expressed at a_{min}/a_{max} (or R_{min}/R_{max}) <0.5. In this case, the dependence is similar to that for the circular and rectangular pipes and can be well approximated with the power function, with the power factor of –3.4.

Hydraulic resistance coefficient increase at the growth in velocity (or at the decrease in viscosity) in bent pipes is associated with essential changes in the velocity distribution owing to convective (transfer) effects – a one- dimensional flow becomes spatial. Rather considerable growth in resistance that occurs in pipes with the small ratio of the cross-section sides (a_{min}/a_{max} or R_{min}/R_{max} << 1) is determined by a great pressure fall within the narrow area of a pipe. At minimum viscosity in pipes with the maximum cross-section variations in the widest place there appear "parasitic" vortices with a counter-stream arising, which also creates an additional hydraulic resistance, however its role is insignificant.

The time required to calculate one flow version based on ANSYS/Flotran software essentially depends on the geometry of the model and the number of finite elements. The duration of calculations is particularly affected by the fluid viscosity – at its decreasing the convergence becomes slower and the time needed for calculations noticeably increases (up to by several times). To do calculations for fluids of lower viscosity it is necessary to form a finer grid of finite elements (especially on the pipe's walls – in the boundary layers), however all this requires much greater computer resource.

The models proposed by the authors allow determination of the hydraulic resistance coefficient and evaluation of its dependence on the fluid viscosity and geometry of the pipes – an essential factor in technological processes connected with fluid or gas transportation.

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DAŽĀDU ĢEOMETRIJAS CAURUĻU
HIDRAULSIKĀS PRETESTBAS SKAITLISKĀ MODELĒŠANA

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K o p s a v i l k u m s

Cauruļvadu hidrauliskā pretestība, ko būtiski ietekmē gan to šķērsgriezuma maiņa garenvirzienā un forma, gan arī caurules liekums, ir būtisks faktors tehnoloģiskajos procesos, kas saistīti ar šķidrumu vai gāzu transportēšanu. Sarežģītas konfigurācijas caurulēm pretestības noteikšanai tādējādi nepieciešama plūsmu skaitliska modelēšana. Šajā darbā novērtēta šādu skaitlisko aprēķinu, lietojot ANSYS/Flotran programmatūru, precizitāte un analizēta cauruļu hidrauliskās pretestības atkarība no to liekuma, šķērsgriezuma formas un tā maiņas garenvirzienā, kā arī šķidruma viskozitātes. Kā bija sagaidāms, mazas viskozitātes (lielu Reinoldsa skaitļu gadījumā) plūsmas pretestība būtiski pieaug gan ar caurules liekumu, gan arī mainoties šķērsgriezuma laukumam garenvirzienā. Savukārt nedeformētās caurulēs hidrauliskā pretestība atbilst analītiski noteiktajām vērtībām.