NUMERICAL MODELING OF HYDRAULIC RESISTANCE IN PIPES OF VARIOUS SHAPES

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Abstract. *Hydraulic resistance of pipes, being essentially affected by shape of their cross-section, is an essential factor in technological processes and in transportation of liquids. For pipes of complicated configuration therefore there is needed numerical modeling. In the present work there's assessed accuracy of numerical calculations by using ANSYS/Flotran software, as well as analyzed there is dependence of hydraulic resistance of pipes upon their size and viscosity. As it could have been expected, resistance of small viscosity flow (in case of high Reynolds numbers) increases as there's increasing the bends of the pipe. It is proved that such methods may be successfully used for analysis of flows also in pipes of complicated configuration, e.g. with longitudinally varying cross-section.*

Keywords: *fluid flow, hydraulic resistance, hydrodynamics, numerical simulation, permeability.*

1. Introduction

In various technological equipments the course of processes and their efficiency is essentially differing upon viscose fluid flows in various types of pipes – one can mention here both chemical and food industry equipment and pipeline systems for transportation of oil products. Also flows in complicated special structures in many cases may be approximately described by using models of pipeline systems – e.g. by analyzing composite material reinforced with fiber beams, technology of impregnation, i.e. saturation with resin [1]. In this case in total the flow of resin is spatial and the through-flow is essentially determined by sizes and shape of gaps between beams of fiber. For a detailed description of such complicated flows there is needed three dimensional mathematical modeling by using Navier-Stokes equations, however, due to capacity of resources of such calculations it is useful to analyze typical structures of flow being established among crossed layers of fiber beams, by approximately considering them as flows in pipes with various cross-section and size, as well as longitudinally varying cross-section area of such pipes. Such a simplified approach in relation to description of complicated flows eases analysis of essential factors affecting flows and is more conveniently applicable for optimization of processes. In the present work there are in succession described simplest flows in pipes with a goal to assess accuracy of numerical calculations and to demonstrate influence of various factors onto the flow.

2. Analytical solutions

By considering fluid flow in the pipe (Fig. 1), its hydraulic resistance coefficient C (1/m^2) in general is defined as follows [2]:

$$C = \frac{S\,\Delta P}{\mu\, L Q}\ , \qquad (1)$$

where S (m^2) is pipe cross-section area, ΔP (Pa) – difference of pressures between its both ends, μ (Pa·s) – fluid viscosity, L (m) – channel length and Q (m^3/s) – throughput. Sometimes in literature used there is also C inverse value – hydraulic permeability K (m^2).

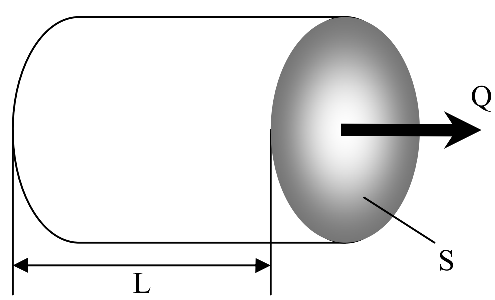

Fig. 1. Pipe element for calculation of hydraulic resistance

In case of certain fluid laminar flow with a given viscosity in a straight pipe, its hydraulic resistance is dependent only upon the cross-section geometry (shape). It is determined by distribution of fluid velocities and therefore dependence of friction force on the pipe walls upon the shape of the cross-section. With a fixed cross-section area S the least hydraulic resistance is for the pipe with a circle section [3]:

$$C = \frac{8}{R^2}, \tag{2}$$

where R (m) is circle radius, for throughput is set by formula:

$$Q = \frac{\pi R^4 \Delta P}{8\mu L}. \tag{3}$$

By considering the pipe with elliptic cross-section with semi-axes a and b in a similar way there are obtained its throughput and hydraulic resistance coefficient expressions [3]:

$$Q = \frac{\pi a^3 b^3 \Delta P}{4\mu L(a^2 + b^2)} \text{ and } C = \frac{4\left(a^2 + b^2\right)}{a^2 b^2}. \tag{4),(5}$$

In case of rectangular cross-section shapes, the flow and resistance calculation expressions become more complicated – throughput capacity for rectangular with sides $2h$ and $2\chi h$, where χ – ratio of side lengths (χ>1) [3]:

$$Q = \frac{\Delta P}{4\mu L}\chi h^4 f(\chi), \tag{6}$$

where $f(\chi) = \frac{16}{3} - \frac{1024}{\pi^5 \chi}\left(\text{th}\frac{\pi\chi}{2} + \frac{1}{9}\text{th}\frac{3\pi\chi}{2} + \dots\right)$ and hydraulic resistance coefficient:

$$C = \frac{16}{h^2}\frac{1}{f(\chi)}. \tag{7}$$

In an approximate calculation one may restrict with two members in $f(\chi)$ expression, and as there increases ratio of cross-section sides, alterations of resistance coefficient become insignificant:

χ	1	2	3	5	10	100	∞
f(χ)	2.253	3.664	4.203	4.665	5.000	5.299	5.333

So, by using expressions (3), (5) un (7), it is analytically possible to calculate values of hydraulic resistance coefficient for pipes with various, simple-shape cross-section. For arbitrary cross-section pipes such a simple analytical calculation is not possible, therefore one of possibilities there is a digital modeling of flows. The said analytical formulas then were used for assessment of accuracy of numerical calculations.

3. Numerical calculations

In the numerical calculations performed there were used model of viscous incompressible liquid and package of programs ANSYS/Flotran [4], where the modeled pipes are sampled by using finite element method. In case of straight pipes from zero there's different only one pipe's longitudinally directed velocity component, but if there's a change of the pipe cross-section in its longitudinal direction, then observed there must be all the three velocity components. By using so acquired distributions of velocities at a fixed difference of pressures in the pipe's ends one may calculate throughput Q.

Since incompressible liquid throughput Q according to liquid incompressibility condition in all cross-sections is equal and in the pipe input velocities are set perpendicular to the surface, then in the throughput calculation there may be used this cross-section area (S) and relevant velocities ($v_\perp$) in the pipe input:

$$Q = Q_{inlet} = \int_{S_i} \vec{v}_i d\vec{S}_i = \int_{inlet} v_\perp dS. \tag{8}$$

Velocities in discrete points are calculated numerically and their relevant finite elements' areas ΔS are approximately equal. In such a case for throughput in the input (and therefore also in the entire pipe) approximately may be calculated by summation of velocities in all n cross-section elements:

$$Q = \int_{ieeja} v_{\perp} dS = \sum v_{\perp} \Delta S = \Delta S \sum v_{\perp} = \frac{S}{n} \sum v_{\perp} \ . \tag{9}$$

Then the hydraulic resistance coefficient (1) simplified numerical calculation formula

$$C = \frac{n \, \Delta P}{\mu L} \frac{1}{\sum v_{\perp}} \tag{10}$$

is conveniently applicable in case of triangular-type elements.

If the pipe input cross-section is rectangular and there be used equal size rectangular elements (cross-section division is regular in the both dimensions), then for throughput calculation by numerically integrating it's easy to use trapeze calculation formula [5], by substituting hydraulic resistances in C (1) expression for throughput Q with integral sum Q_{num}:

$$\begin{aligned} Q_{num} = h \cdot g \Bigg\{ & \left[\frac{v_{\perp}(x_0, y_0)}{4} + \sum_{j=1}^{M-1} \frac{v_{\perp}(x_0, y_j)}{2} + \frac{v_{\perp}(x_0, y_M)}{4} \right] + \\ & \left[\frac{v_{\perp}(x_N, y_0)}{4} + \sum_{j=1}^{M-1} \frac{v_{\perp}(x_N, y_j)}{2} + \frac{v_{\perp}(x_N, y_M)}{4} \right] + \\ & \sum_{i=1}^{N-1} \left[\frac{v_{\perp}(x_i, y_0)}{2} + \sum_{j=1}^{M-1} v_{\perp}(x_i, y_j) + \frac{v_{\perp}(x_i, y_M)}{2} \right] \Bigg\} \end{aligned} \ , \tag{11}$$

where h and g (Fig. 2) are lengths of element sides.

Velocities in junctions with minimal or maximal values of coordinates x or y on the pipe walls due to so-called adhesion is equal to zero, therefore the expression (11) gets simplified:

$$Q_{num} = h \cdot g \sum_{i=1}^{N-1} \left(\sum_{j=1}^{M-1} v_{\perp}(x_i, y_j) \right). \tag{12}$$

By using such type numerical calculation expressions there are obtained fluid throughput approximated values in pipes of various shapes. Firstly there's verified accuracy of numerical modeling results in straight pipes with a simple cross-section shape, by comparing the results obtained with analytical ones. After there's numerically modeled flow in pipes with various bend radius and different liquids viscosities of which differ.

4. Influence of the pipe cross-section shape

For comparison of numerical calculation results with theoretical ones there were selected 3 versions of pipes cross-sections at equal ratio of cross-section area – circle, square and rectangular with a side χ=9 (Fig. 3). Depending upon the shape of the selected pipe cross-section there are used two-type finite elements – triangular-type elements for circle and square-type elements for square and rectangular. Geometric data used in the model and physical characteristic values of liquids are summarized in Table 1.

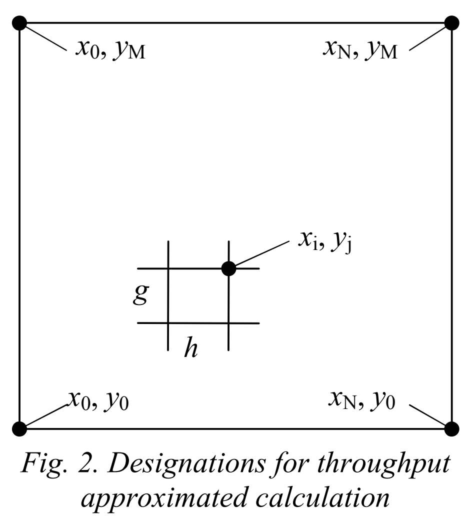

Fig. 2. Designations for throughput approximated calculation

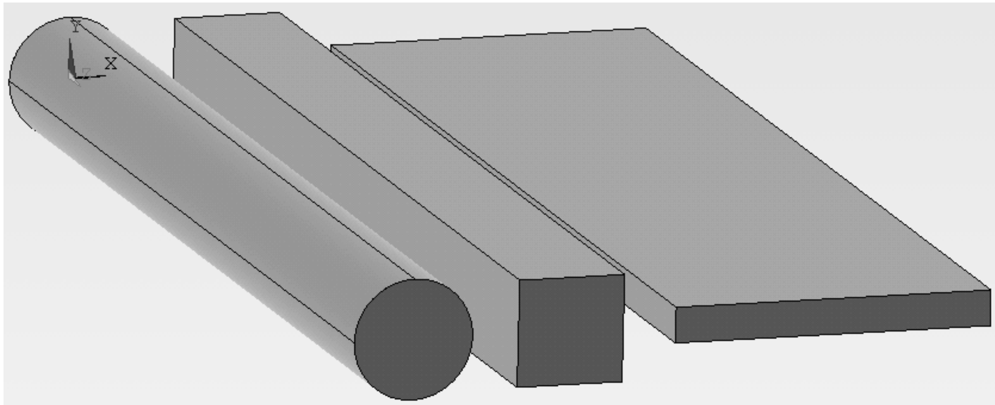

Fig. 3. Pipes with various cross-section used in calculations

Table 1. Geometric and physical parameters of the model

Item	**Designation**	**Value**	**Unit**
Cross-section area	S	0.01	m^2
Radius (for circle)	R	0.0564	m
Side (for square)	d	0.1	m
Shortest side (for parallelogram)	a	0.0333	m
Longest side (for parallelogram)	b	0.3	m
Pipe length	l	1	m
Difference of pressures	ΔP	10	Pa
Density	ρ	1000	kg/m^3
Dynamic viscosity	μ	1	Pa·s

Comparison of calculation results with theoretical ones is given in Table 2, but their graphical interpretation – in Fig. 4. As one can see, numerical calculations do well accord with analytical ones. However, accuracy of numerical results is essentially dependent upon calculation method of the selected pipe – for rectangulars more precise there is numerical integration formula (11), what being not applicable for circle due to its shape. Therefore for it calculation is effected by using only formula (9).

Table 2. Analytical and numerical results in pipes with various shapes of cross-sections

Cross-section shape	Type	Number of elements (n)	Characteristic element's area (m^2)	$C_{analytical}$ ($1/m^2$)	$C_{numerical,\,1}$ *acc.*(*9*) ($1/m^2$)	$C_{numerical,\,2}$ *acc.*(*12*) ($1/m^2$)
○	∇	7185	$1.4\text{-}1.5 \cdot 10^{-6}$	2515	2783	not used
□	□	6561		2804	3027	2953
▭	□	6748		11664	12134	11663

The error between numeric calculation results and relevant analytical solutions does not exceed 7%, therefore one can regard that numerical solutions do sufficiently well describe the process and they may be used in calculations of other pipe types, where no analytical calculations are possible, e.g. for bent pipes and for pipes with longitudinally altering cross-section.

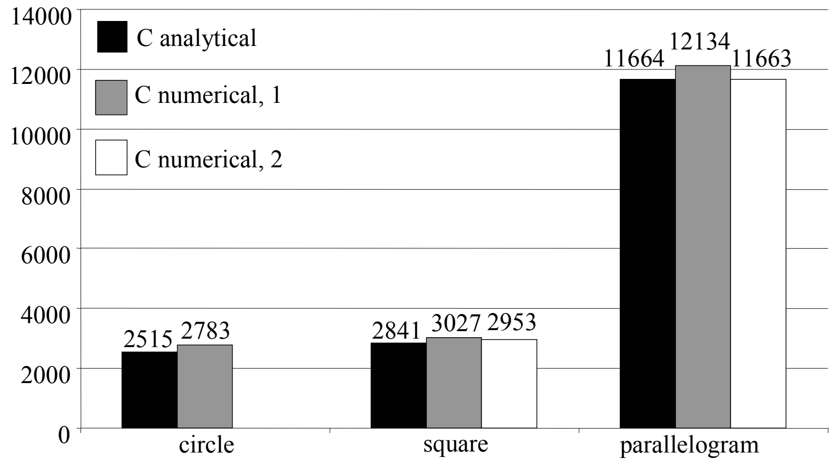

Fig. 4. Comparison of analytical and numerical results in pipes with various shape of cross-section

5. Analysis of influence of the pipe bend and viscosity

Let's view pipes with an equal square-type cross-section (0,1 m×0,1 m) and an equal length, but various bends – starting with a straight pipe and ending up with a span changing the flow direction by 180° (Fig. 5). The bends of the 7 viewed pipes are accordingly: 0; π/4; π/2; π/1.75; π/1.5; π/1.25 and π (1/m). In these pipes modeled there is flow of fluids with various viscosities, allowing to determine dependence of hydraulic resistance not only upon their bend, but also the velocity of the flow or Reynolds number:

$$\mathrm{Re} = \frac{v\,\rho\,d}{\mu}, \tag{13}$$

where v (m/s) is an average flow velocity in the pipe cross-section (is calculated upon distribution of the velocity in the pipe input) and d (m) is a characteristic length scale in the cross-section direction (in this case it is assumed equal with the length of the square's side).

In calculations used there is a series of dynamic viscosity values as follows: μ=1; 0.1; 0.05; 0.035; 0.02; 0.015 (Pa·s). Other data correspond to Table 1. For numerical integration of the fluid flow there was used trapeze formula (12), for it ensures a higher accuracy for rectangular-type cross-sections.

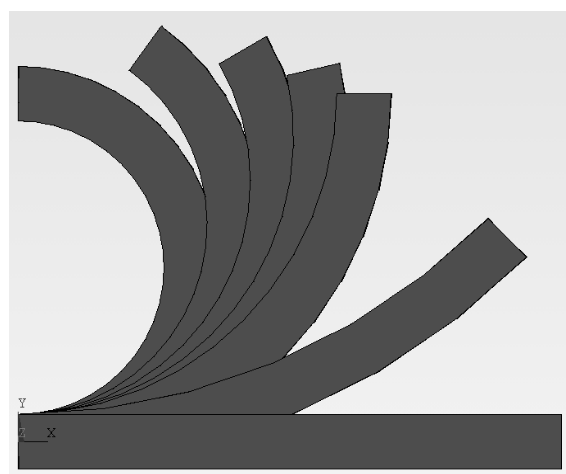

Fig. 5. Pipes with various bends

Results of numerical calculations depending upon viscosity of liquid (Reynolds number) and the bend of the pipe are summarized in Table 3. As one can see, at small Reynolds numbers, i.e. in case of a slow flow, hydraulic resistance coefficient values are growing little as increasing there is the bend of the pipe. Whereas in case of a relatively quick (although also laminar) flow at Re=700 resistance coefficient in the pipe with a 180^{o} bend is 2 times greater than in the straight pipe at the same cross-section. It is related to an increase of impulse convective transfer role in comparison with viscous effects, as the velocity increases (Fig. 6). In the straight pipe with constant cross-section $(\vec{v}\cdot\nabla)\vec{v} = 0$ independent of the flow velocity, therefore there hydraulic resistance is not dependent upon μ. Since in calculations used there is a laminar flow model, then flows at smaller viscosities (Re>1000) are not viewed, for as the velocity increases, they become unstable and turbulent.

Table 3. Results of hydraulic resistance calculations at various viscosities and bends

Bend k	μ=1	μ=0,1	μ=0,07	μ=0,05	μ=0,35	μ=0,02	μ=0,015
	Re=0,3	Re=30	Re=60	Re=100	Re=170	Re=420	Re=710
	Hydraulic resistance C (1/m^2)						
π	2902	3034	3309	3757	4338	5398	5741
π/1,25	2902	2978	3191	3570	4071	4946	5237
π/1,5	2903	2945	3115	3444	3891	4631	4802
π/1,75	2903	2924	3063	3353	3759	4405	4595
π/2	2902	2909	3025	3282	3653	4224	4409
π/4	2902	2871	2912	3029	3246	3564	3665
0	2903	2853	2856	2856	2856	2856	2856

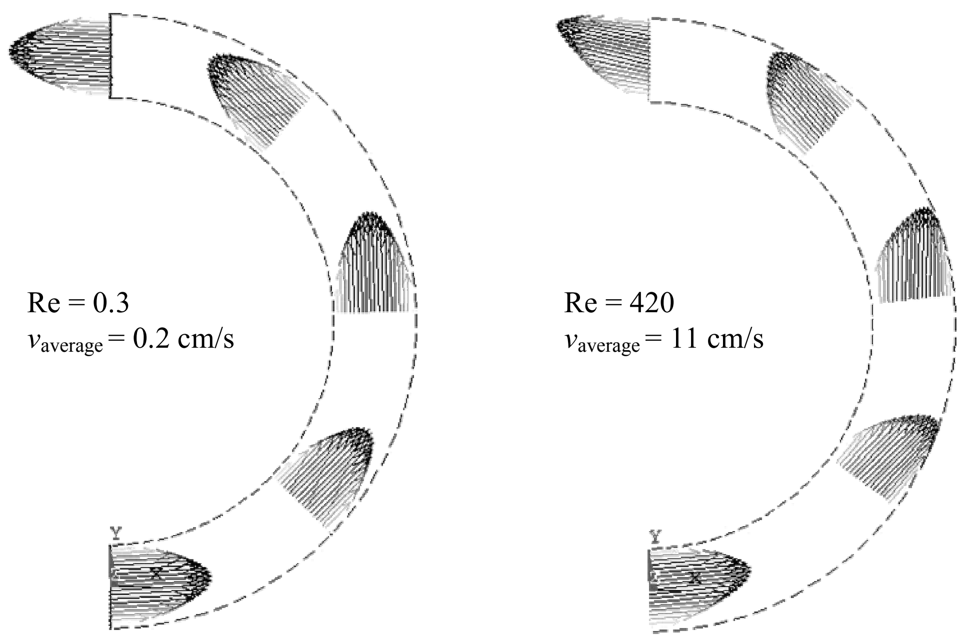

Fig. 6. Distribution of velocity in cross-sections of a pipe at various values of Re

Graphically dependence of hydraulic resistance coefficient upon bends is shown in Fig. 7. As one can see, an increase of this resistance is to be expressed by liner function: $C=a \cdot k+C_o$, where coefficient C_o is resistance of the straight pipe being determined only by its cross-section shape, but a is the function of viscosity or Re. Therefore hydraulic resistance is linearly dependent upon the bend of the pipe.

Hydraulic resistance dependence upon Reynolds number inverse value (1/Re) is vividly shown in Fig. 8: hydraulic resistance is rapidly decreasing from high values at 1/Re≈0 up to hydraulic resistance value C_o, what corresponds to the straight pipe. As aforesaid, in pipes with a great bend dependence from Re is clearly expressed, while in its turn in pipes with a smaller bend this dependence is not essential and for a straight pipe hydraulic resistance coefficient is not dependent upon Reynolds number (or viscosity).

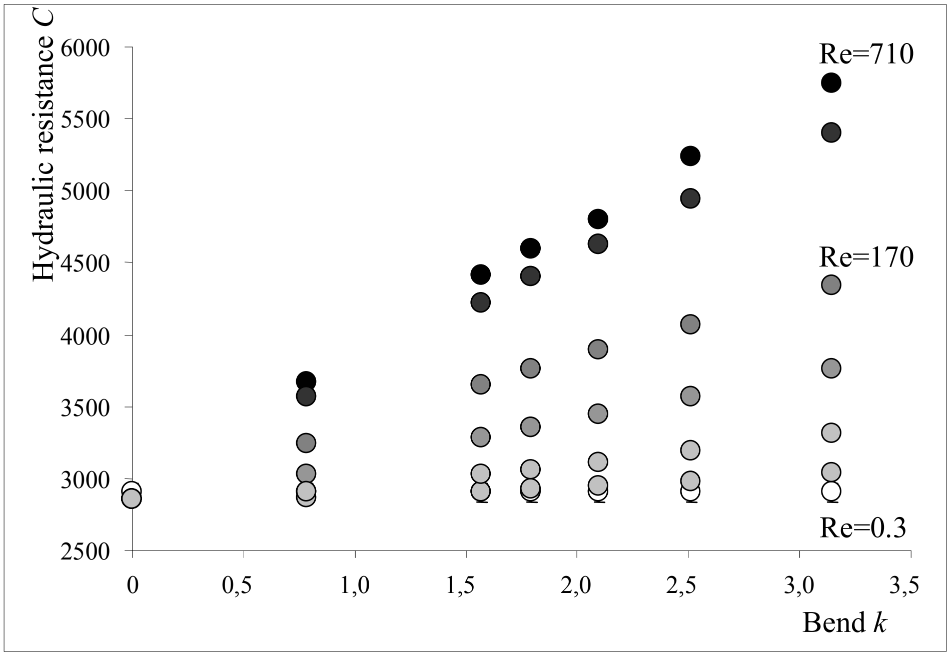

Fig. 7. Hydraulic resistance dependence upon the bend

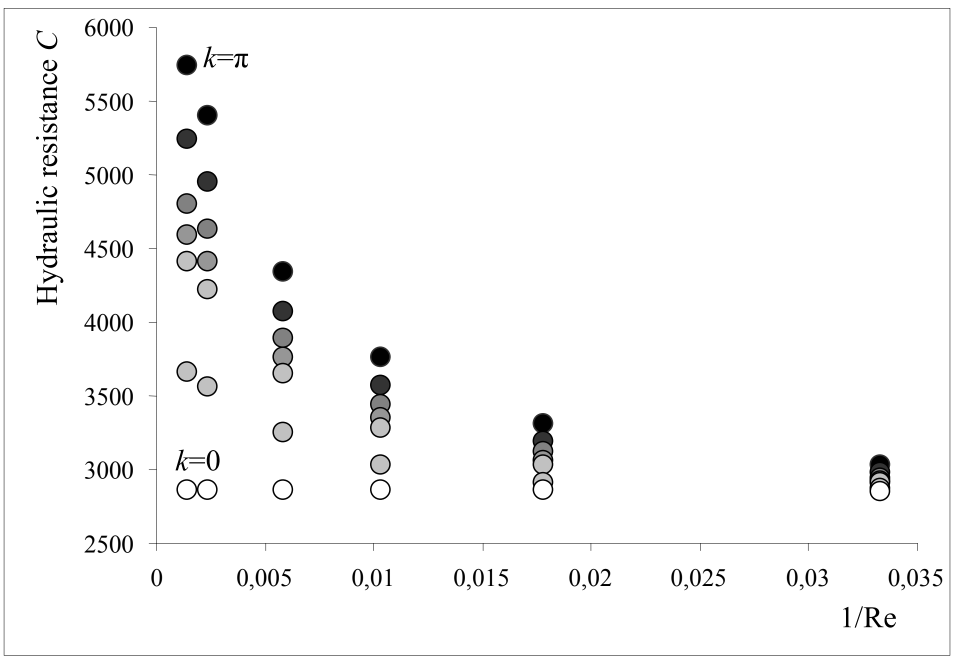

Fig. 8. Hydraulic resistance dependence upon inverse Reynolds number

6. Conclusions

Numeric calculations of viscous fluid laminar flow in pipes with various cross-section shapes, by using ANSYS/Flotran software, proved a good accuracy of results, simple form for which there exist analytical solutions. Calculations of flows, made in continuance of the work, in bent pipes allowed to determine hydraulic resistance increase jointly with an increase of the bend and the characteristic Reynolds number value. By using this approved approach, further it's envisaged to effect hydraulic resistance analysis in bent pipes with a complicated cross-section shape, as well as in straight pipes, cross-section of which into their longitudinal direction being variable.

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