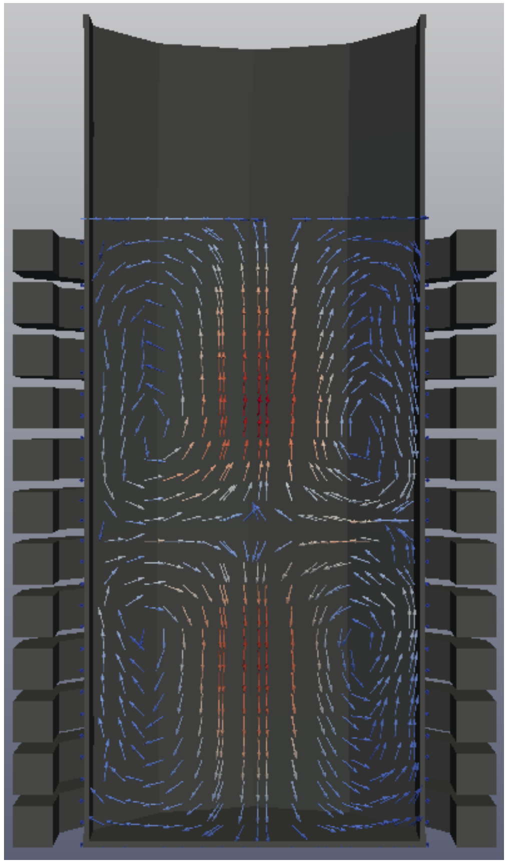


Connecting OpenFOAM with an external electromagnetic FDM solver for magnetofluiddynamic simulations

P. Turewicz, M. Bibani, E. Baake

Contents

Introduction

Introduction

- **Background**

  - OpenFOAM is a powerful tool for solving partial differential equations (PDEs), but undeveloped for complex fields

  - Solutions: -Enhancement of integrated complex libraries and development of a new MFD-solver

    -Coupling an external EM-solver with an integrated fluid flow solver

- **Objective**

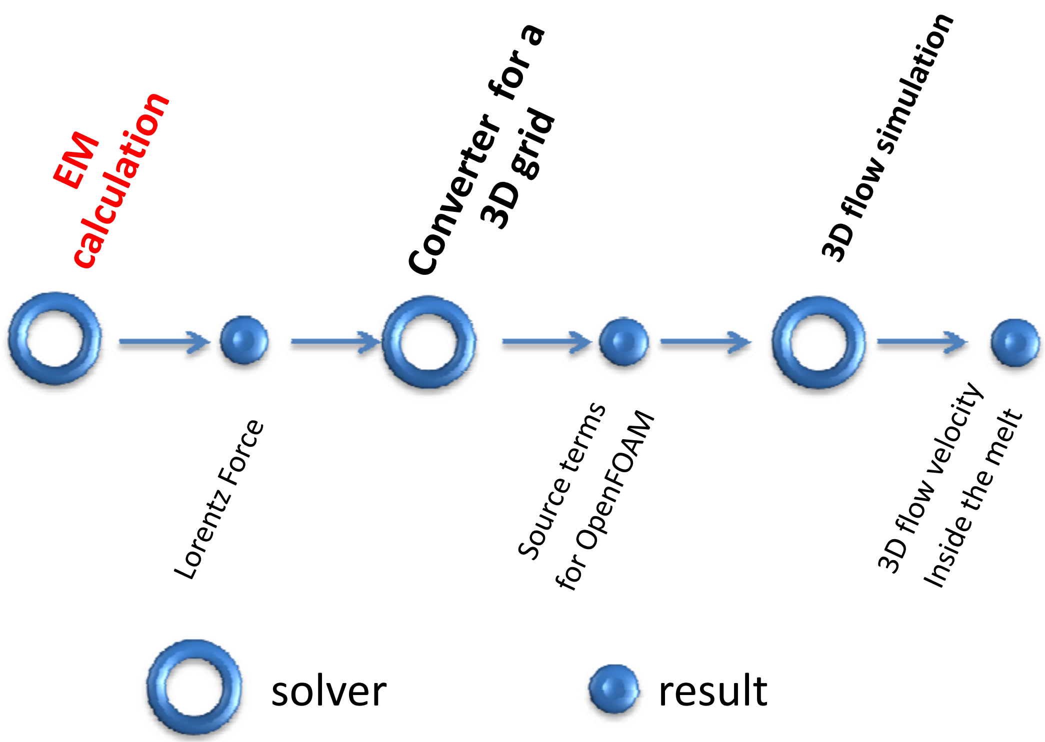
  - External solver based on finite difference method (FDM) for electromagnetic field calculation

  - Coupling EM source terms with a suitable integrated solver and calculation of the flow

  - Verification by experimental data

Introduction

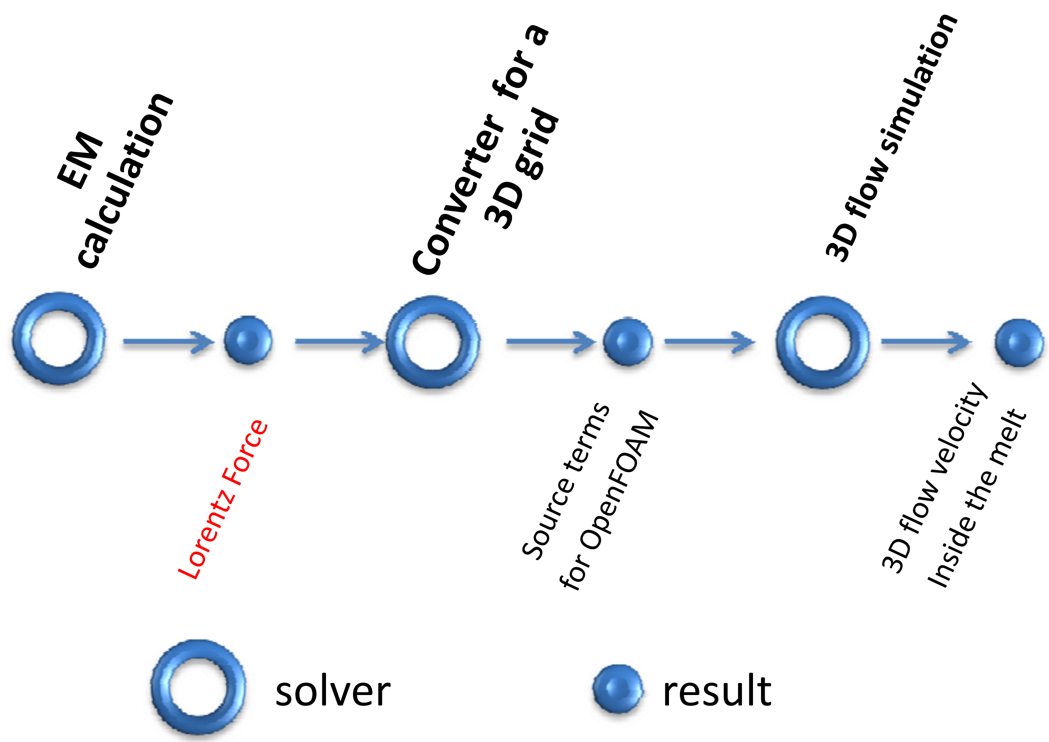
- **Procedure**
  - Generation of a FDM grid, applying BC for the vector potential and solving the EM problem



Introduction

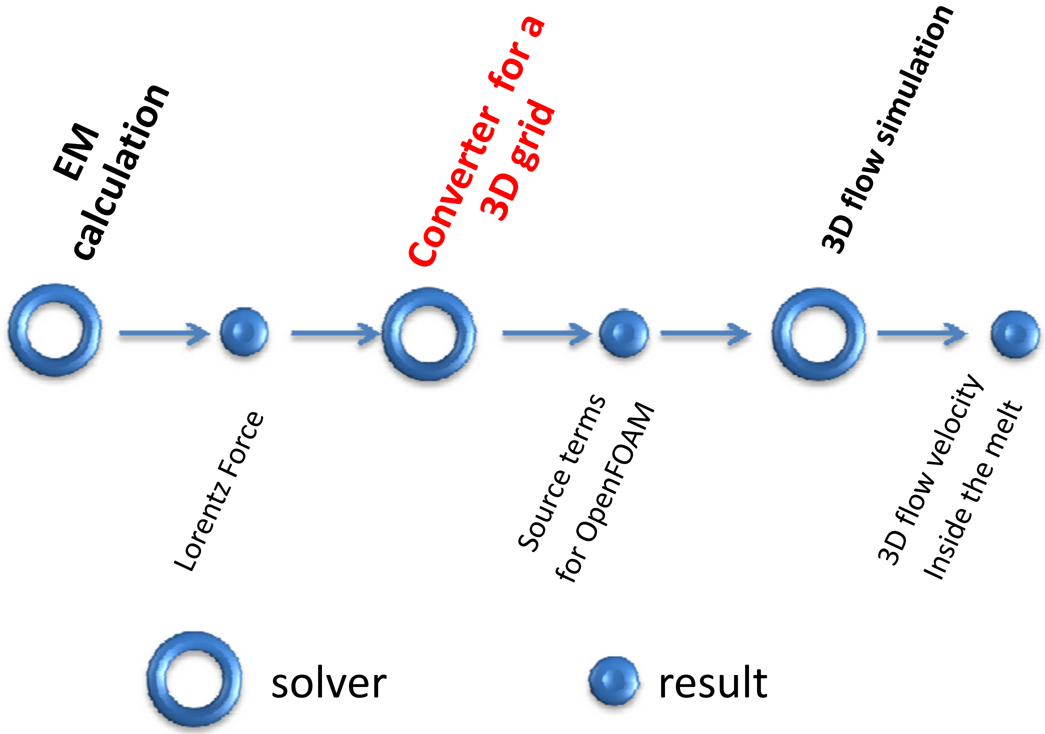
➢ **Procedure**

- Generation of a FDM grid, applying BC for the vector potential and solving the EM problem
- Calculation of the source terms



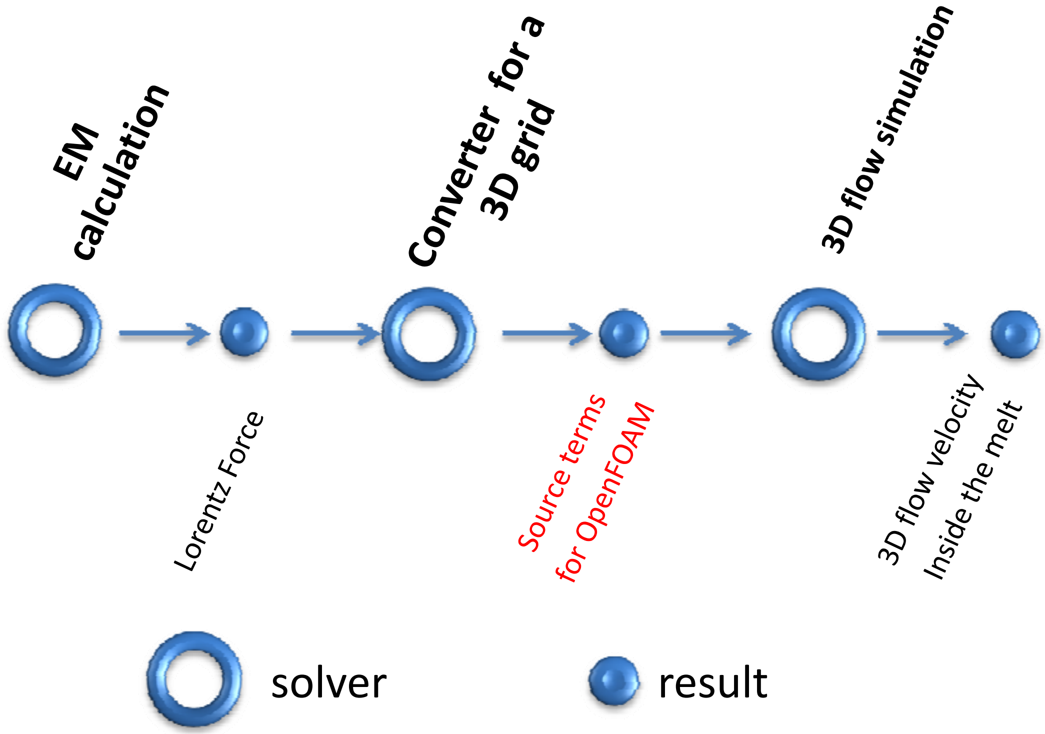
Introduction

- **Procedure**
  - Generation of a FDM grid, applying BC for the vector potential and solving the EM problem
  - Calculation of the Lorentz force
  - Converting the Grid



Introduction

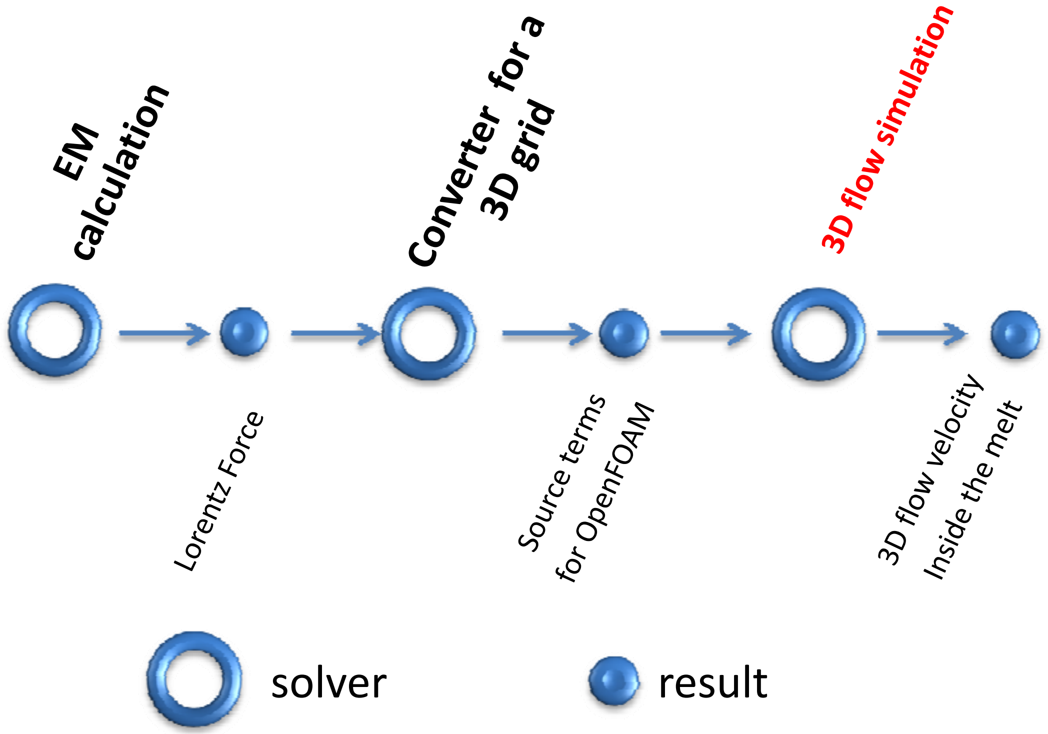
- **Procedure**
  - Generation of a FDM grid, applying BC for the vector potential and solving the EM problem
  - Calculation of the Lorentz force
  - Converting the Grid
  - Integrating the source terms



Introduction

➢ **Procedure**

- Generation of a FDM grid, applying BC for the vector potential and solving the EM problem
- Calculation of the Lorentz force
- Converting the Grid
- Integrating the source terms
- Using the buoyant-BoussinesqSimpleFoam to calculate the flow



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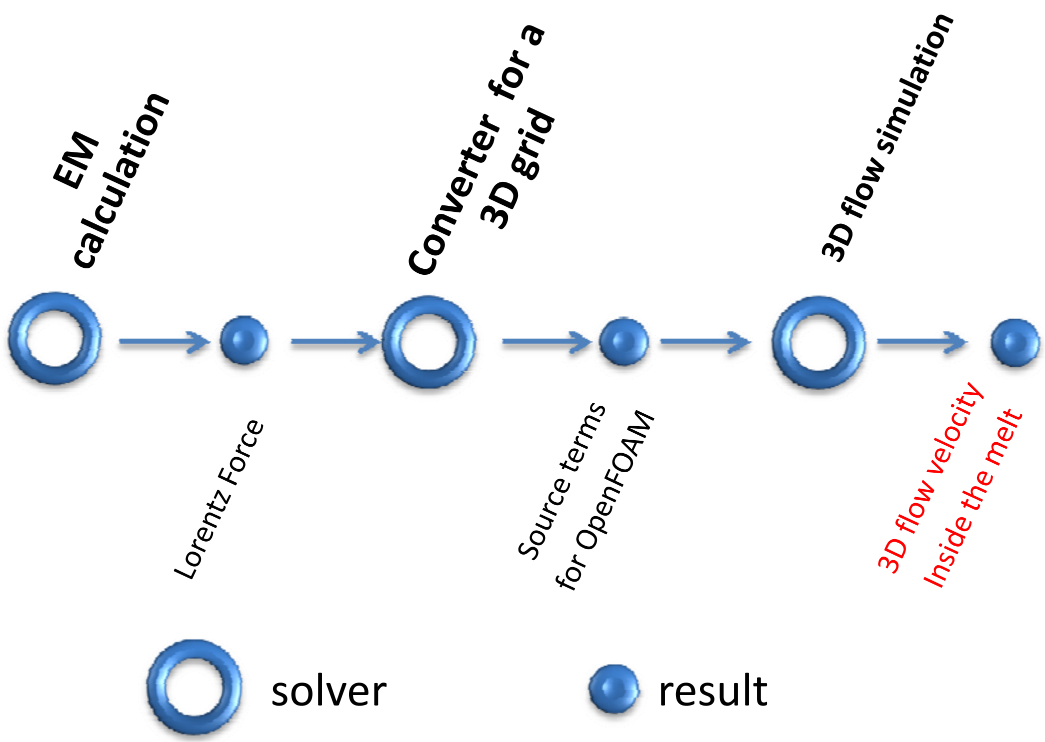
- Generation of a FDM grid, applying BC for the vector potential and solving the EM problem
- Calculation of the Lorentz force
- Converting the Grid
- Integrating the source terms
- Using the buoyant-

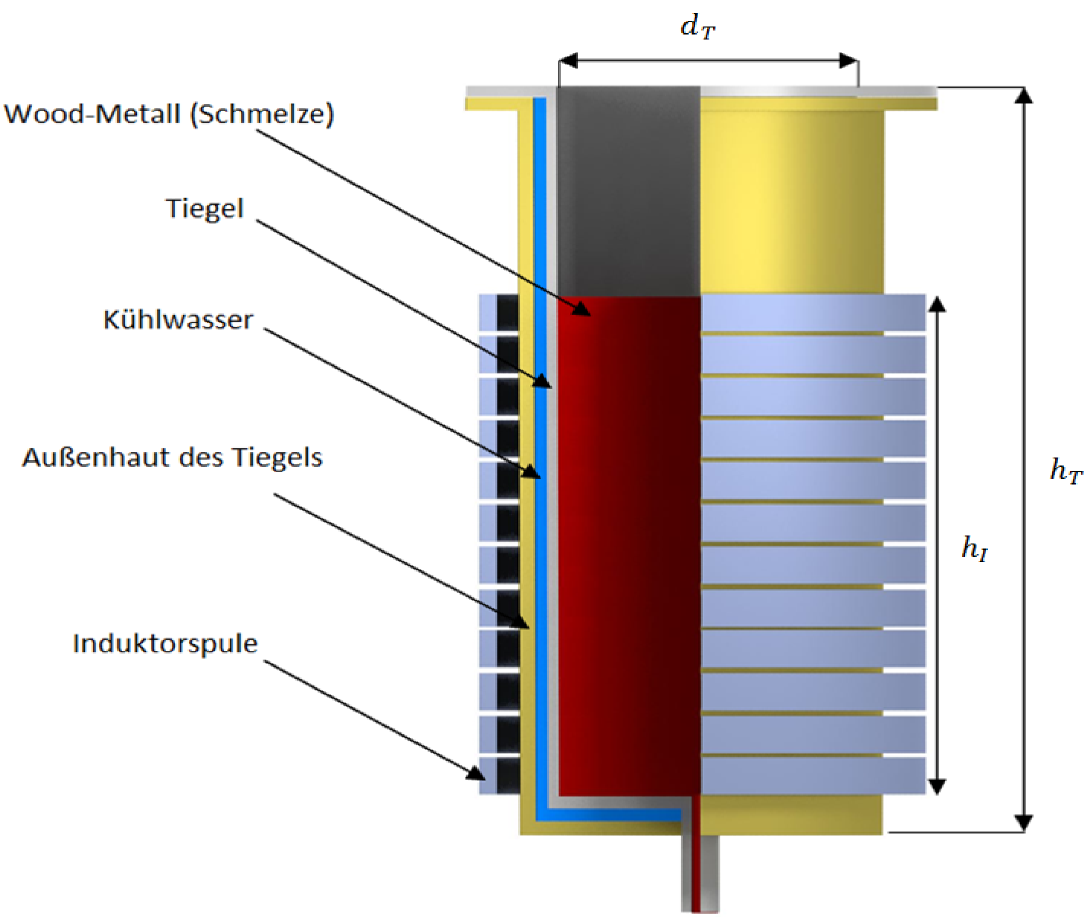
  BoussinesqSimpleFoam

  to calculate the flow
- Evaluate and visualize the results



Modeling

The Installation



$d_T = 316$ mm
$h_I = 570$ mm
$h_T = 756$ mm

- A cylindrical induction crucible furnace
- Inductor: N = 12 windings, I = 2 KA and f = 400 Hz, δ = 3,5mm
- Crucible wall thickness d = 0,8 mm
- Wood´s metal melt (Ba1994):
  - 74 °C melting temperature
  - density $9.4 \cdot 10^{3} \frac{kg}{m^{3}}$
  - Electr. Conductivity $10^{6} \frac{S}{m}$
  - Heat capacity $168 \frac{Ws}{Kg \cdot K}$
  - Kin. Viscosity $0.45 \cdot 10^{-6} \frac{m^2}{s}$

Electromagnetic Field

- Taking into account the axisymmetrical geometry of the installation, the problem is solved in 2D
- Applied boundary conditions:

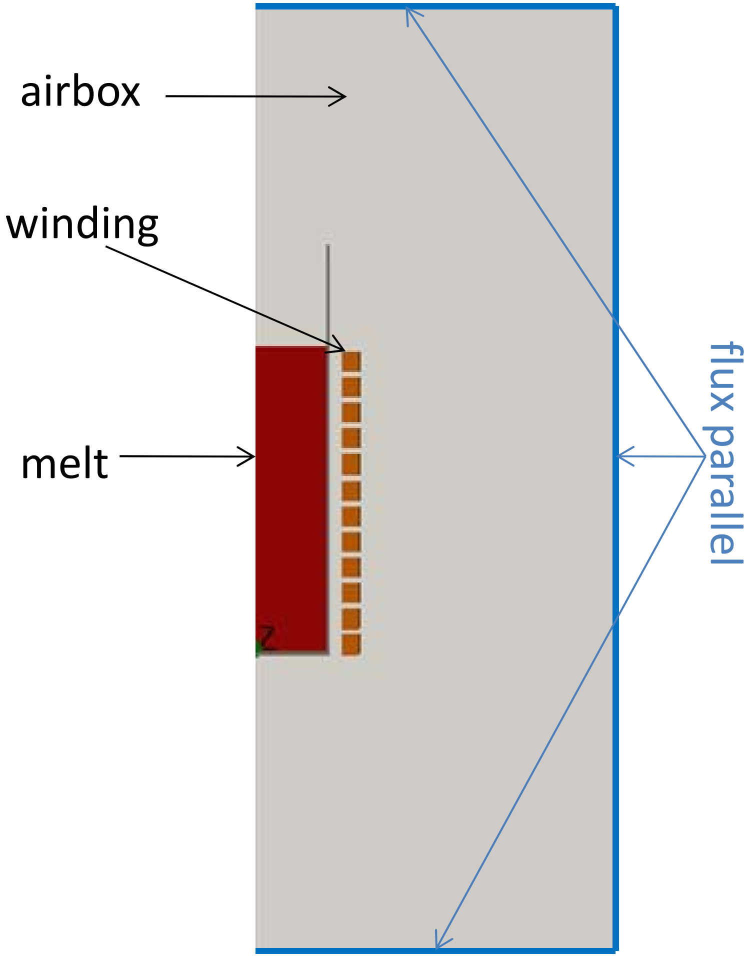
  – flux parallel at the boundaries

  – reduction of the windings to the penetration depth and applying J=12 A/mm² on inner wall of each coil tube

  – induction in the crucible walls is ignored

  – using constant material properties

  – considering a plane free surface



Electromagnetic Field

➢ Mathematical Background

axisymmetry

- Vector potential:

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial rA}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 A}{\partial \varphi^2}+\frac{\partial^2 A}{\partial z^2}-j\omega\mu\kappa\, A=-\mu S$$

- After solving the vector potential the current density can be obtained by using

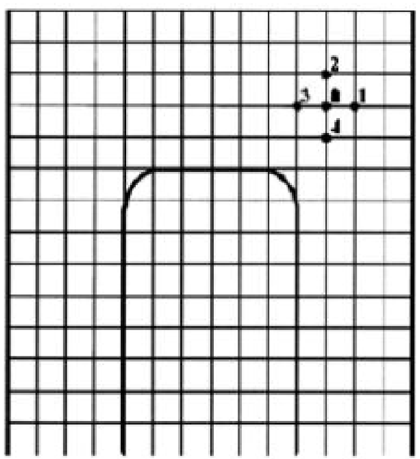
$$B=\nabla\times A \qquad \text{and} \qquad J=\nabla\times\left(\frac{1}{\mu}B\right)$$

- Finally the Lorentz force can be computed with

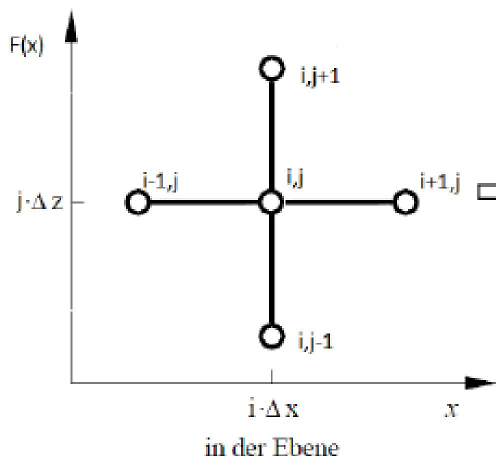
$$f_{EM}=J\times B=\nabla\times\left(\frac{1}{\mu}B\right)\times B$$

Electromagnetic Field

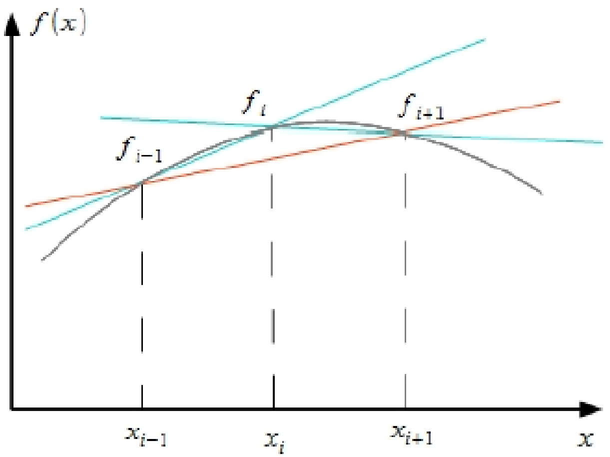
Solving the equations by using the Finite-Difference-Method



(a) Numerical Grid



(b) Nodes



(c) Approximation of the equations

Electromagnetic Field

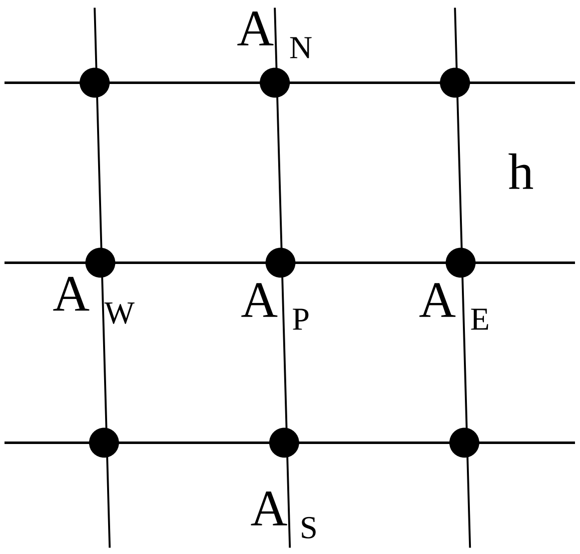
$$\left.\frac{\partial A}{\partial r}\right|_P = \frac{A_E - A_W}{2h}$$

mesh size

$$\left.\frac{\partial A}{\partial y}\right|_P = \frac{A_N - A_S}{2h}$$

$$\Rightarrow \text{div } A = \frac{A_N - A_S}{2h} + \frac{A_E - A_W}{2h}$$

$$\Rightarrow \Delta A = \frac{A_W + A_E + A_N + A_S - 4A_P}{h^2}$$

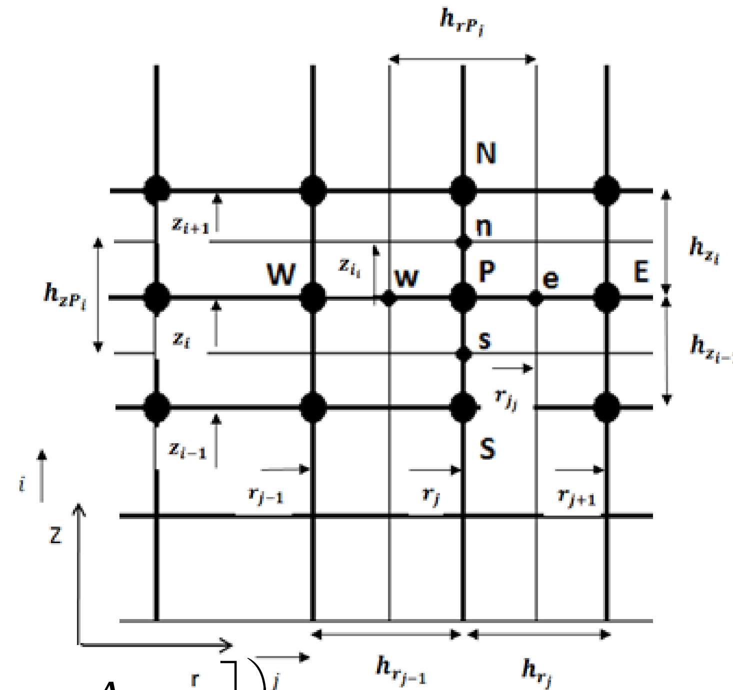


Imprecise for non uniform grids

Electromagnetic Field

- Result improvement using the principle of the staggered grid (Prof. Dr.-Ing. O. Pesteanu)

$$\underbrace{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rA)}{\partial r}\right]}_{1} + \underbrace{\frac{\partial^2 A}{\partial z^2}}_{2} - j\omega\mu\kappa\, A = -\mu_0 S$$



1: $$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rA)}{\partial r}\right]\Bigg|_P = \frac{\partial F}{\partial r}\Bigg|_P \approx \frac{F_e - F_w}{hrP_j}$$

$$= \frac{1}{hrP_j}\left(\left[\frac{1}{r_{j_j}}\frac{r_{j+1}A_{i,j+1} - r_j A_{i,j}}{h_{r_j}}\right] - \left[\frac{1}{r_{j_{j-1}}}\frac{r_j A_{i,j} - r_{j-1}A_{i,j-1}}{h_{r_{j-1}}}\right]\right)$$

2: $$\frac{\partial^2 A}{\partial z^2}\Bigg|_P = \frac{1}{h_{zP\,i}}\left(\frac{\partial A}{\partial z}\Bigg|_n - \frac{\partial A}{\partial z}\Bigg|_s\right) = \frac{1}{h_{zP\,i}}\left(\frac{A_{i+1,j} - A_{i,j}}{h_{z\,i}} - \frac{A_{i,j} - A_{i-1,j}}{h_{z\,i-1}}\right)$$

Electromagnetic Field

- After calculating the vector potential, the magnetic induction can be obtained using:

$$B = rot\ A = \left(\frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)e_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)e_\varphi + \frac{1}{r}\left(\frac{\partial (rA_\varphi)}{\partial r} - \frac{\partial A_r}{\partial \varphi}\right)e_z$$

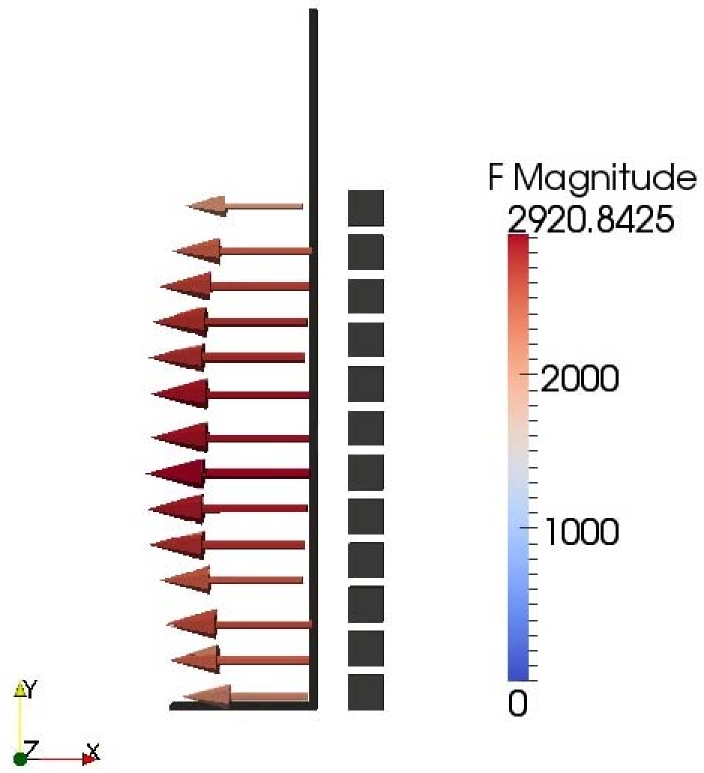
$$= \underbrace{\left(-\frac{\partial A_\varphi}{\partial z}\right)}_{1} e_r + \frac{1}{r}\underbrace{\left(\frac{\partial (rA_\varphi)}{\partial r}\right)}_{2} e_z$$

$$1: \quad -\frac{\partial A_\varphi}{\partial z}\bigg|_P e_r = -\frac{1}{h_{zP\,i}}\left(\frac{h_{z_{i-1}}}{2}\frac{A_{i+1,j} - A_{i,j}}{h_{z\,i}} + \frac{h_{z_i}}{2}\frac{A_{i,j} - A_{i-1,j}}{h_{z\,i-1}}\right)$$

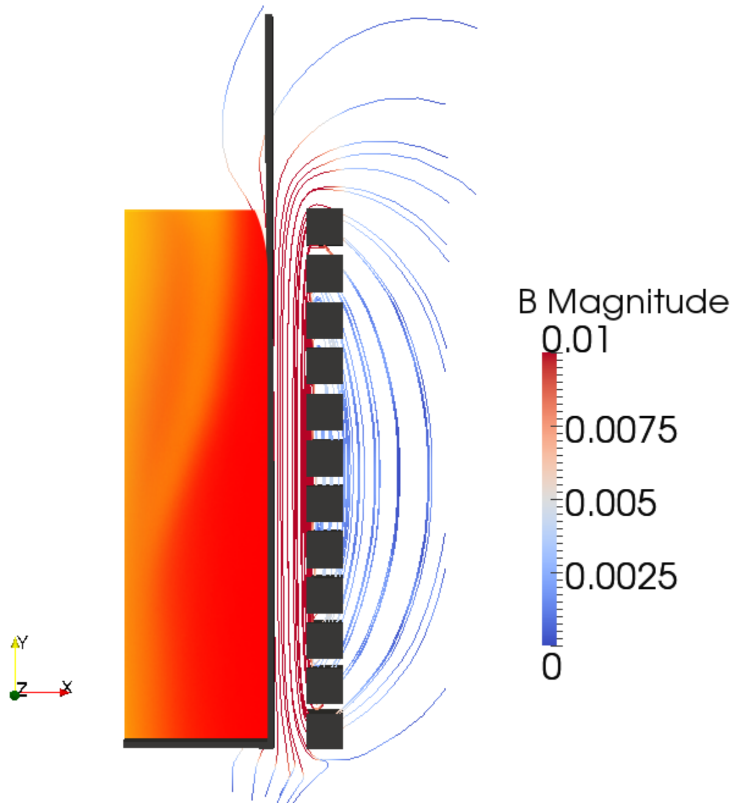
$$2: \left[\frac{1}{r}\frac{\partial (rA_\varphi)}{\partial r}\right]\bigg|_P = -\frac{1}{h_{rP\ j}}\left(\left(\frac{h_{r_{j-1}}}{2}\frac{1}{r_{j_j}}\frac{r_{j+1}A_{i,j+1} - r_jA_{i,j}}{h_{r\ j}}\right) + \left[\frac{h_{r_j}}{2}\frac{1}{r_{j_{j-1}}}\frac{r_jA_{i,j} - r_{j-1}A_{i,j-1}}{h_{r\ j-1}}\right]\right)$$

- Similar procedure for the current density

Electromagnetic Field



Force density [N/m³]



magnetic induction [T/m³]

Modeling of the flow field

Flow Field

➢ **Selection of a suitable solver in OpenFOAM**

▪**BuoyantBoussinesqSimpleFOAM**

- ✓ Steady state
- ✓ Turbulence
  (k ε – model)
- ✓ incompressible
- – ~~Buoyancy~~
- – ~~Temperature~~

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} = \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu\left(\frac{\partial^2 v_i}{\partial x_j \partial x_j}\right) + f_i - \rho_0 \alpha (T - T_0) g_i, i = 1,2,3$$

v	-	velocity
p	-	pressure
f	-	Lorentz force
ρ_0	-	Density at Temperature T_0
α	-	Expansion coefficient
ν	-	Kinematic viscosity

Flow Field

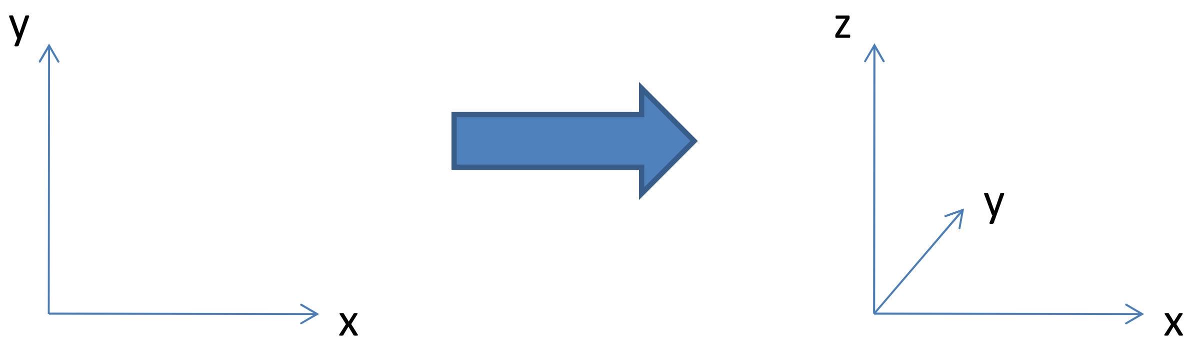
➢ **Convertion of the Lorentz force from 2D to 3D**

f - Lorentz force in 2D

F - Converted Lorentz force in 3D

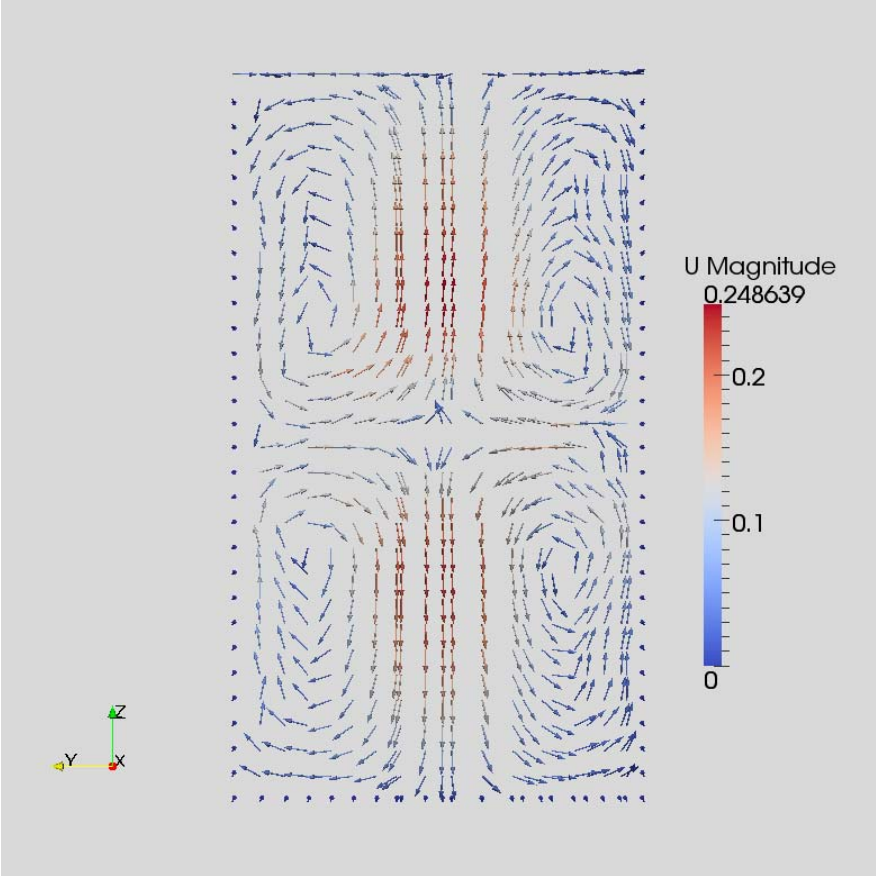
$$r = \sqrt{x^2 + y^2}$$

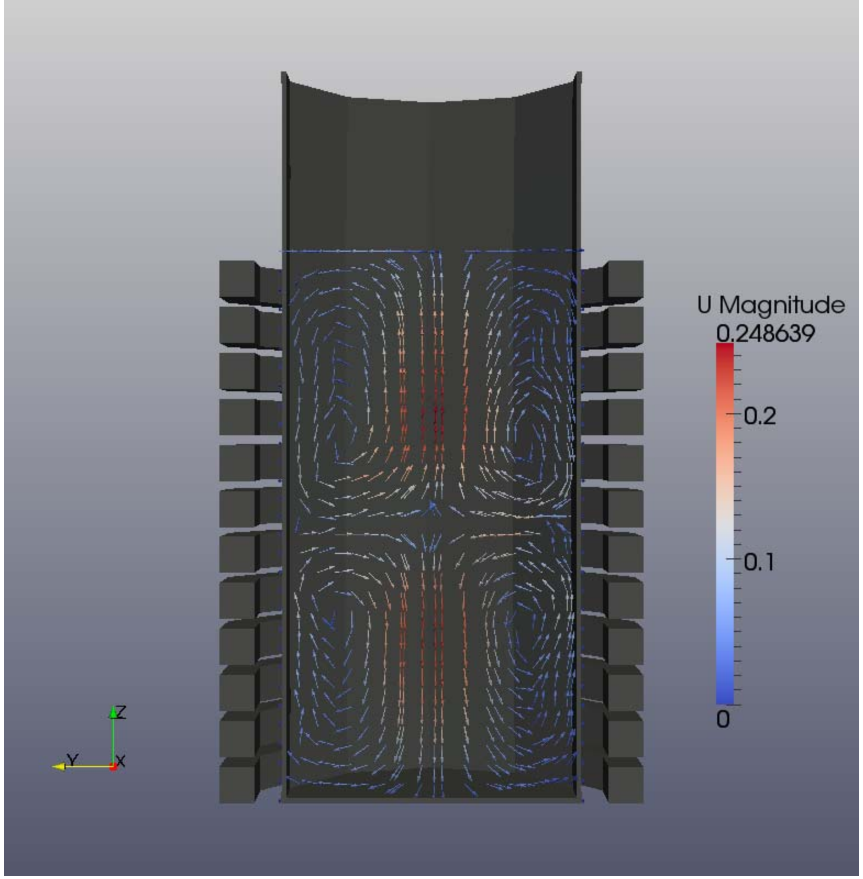
$$F_x = \frac{f_x \cdot x}{r} \qquad F_y = \frac{f_y \cdot y}{r} \qquad F_z = f_y$$



Results

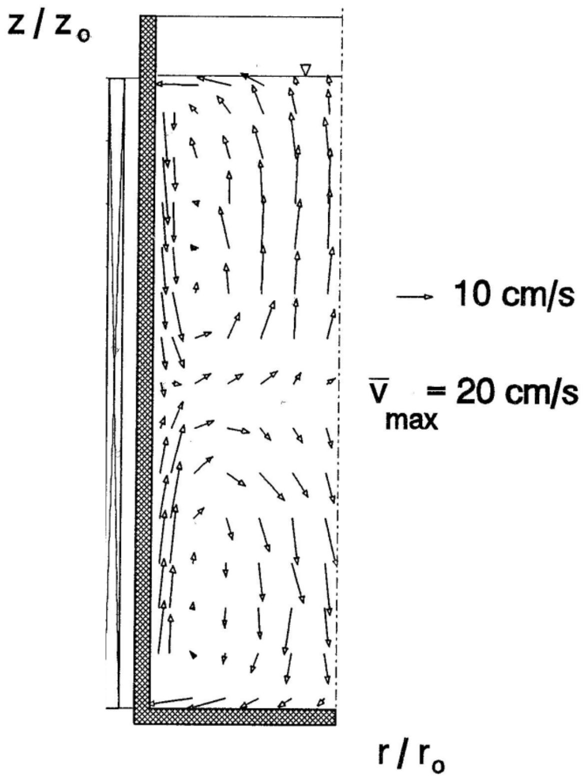
Results



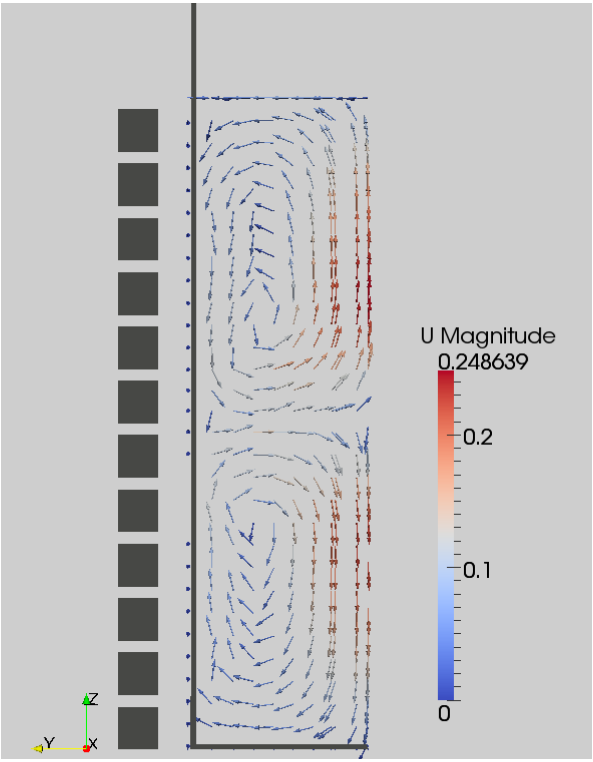


Calculated flow velocities in OpenFOAM [m/s]

Results



Time averaged measured flow velocity [m/s] (Baake1994)



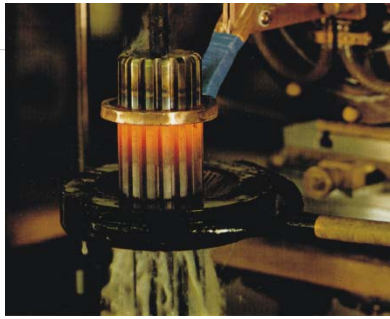
Calculated flow velocities in OpenFOAM [m/s]

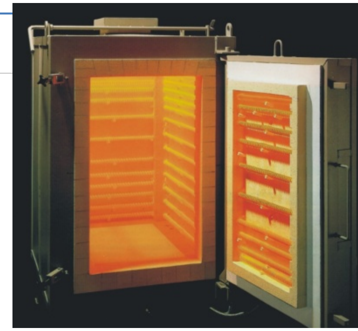
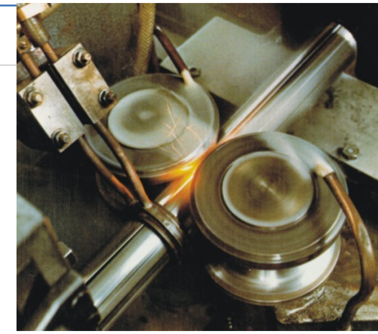
Conclusion and Outlook

Conclusion and Outlook

- **Summary:**
  - An external FDM solver for the electromagnetic field was created
  - Coupling of the solver with OpenFOAM
  - The results have been validated with experimental data

- **Outlook:**
  - Enhancement of integrated complex objects to solve (partial differential equations) PDEs for complex Fields



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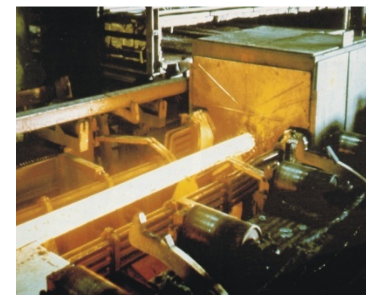


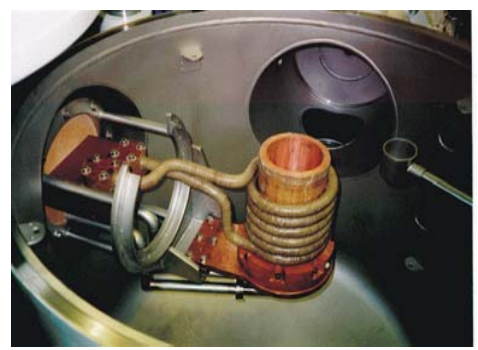
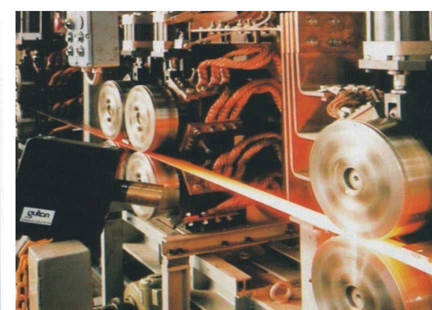

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